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on Early Childhood
Development



Numeracy

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Table of content

Synthesis 4

Numerical Knowledge in Early Childhood 7
CATHERINE SOPHIAN, PHD, JUNE 2009

Early Predictors of Mathematics Achievement and Mathematics Learning Difficulties 13
[1]NANCY C. JORDAN, PHD, [2]BRIANNA L. DEVLIN, PHD, DECEMBER 2023

Early Numeracy: The Transition from Infancy to Early Childhood 22
KELLY S. MIX, PHD, DECEMBER 2023

Learning Trajectories in Early Mathematics – Sequences of Acquisition and Teaching 38
DOUGLAS H. CLEMENTS, PHD, JULIE SARAMA, PHD, AUGUST 2023

Fostering Early Numeracy in Preschool and Kindergarten 45
ARTHUR J. BAROODY, PHD, APRIL 2024

Mathematics Instruction for Preschoolers 63
JODY L. SHERMAN-LEVOS, PHD, JULY 2010

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Synthesis

How important is it?

Numeracy is sometimes defined as understanding how numbers represent specific magnitudes. This understanding is reflected in a variety of skills and knowledge (ex. counting, distinguishing between sets of unequal quantities, operations such as addition and subtraction), and so numeracy often is used to refer to a wide range of number-related concepts and skills. These abilities often emerge in some form well before school entry. The idea of exposing young children to Early Childhood Mathematical Education (ECME) has been around for more than a century, but current discussions revolve around the goals of early training in numeracy and the methods by which these goals should be achieved. Early mathematical learning can and should be integrated in children's everyday activities through encounters with patterns, quantity, and space. Giving children ample and developmentally appropriate opportunities to practice their skills in mathematics, can strengthen the link between children's early abilities in mathematics and the acquisition of mathematical knowledge in school. Unfortunately, children do not all have an equal chance to exercise these skills, hence the importance of ECME. Research on numeracy and early mathematical skills is important to formulate the program and objectives of ECME.

Difficulties in mathematics are relatively common among school-age children. Approximately 1 in 10 children will be diagnosed with a learning disorder related to mathematics during their education. One of the most severe forms is developmental dyscalculia, which refers to an inability to count and tally collections of items and to distinguish numbers from one another.

What do we know?

Basic mathematical knowledge emerges in infancy and may be shaped by exposure to verbal numeracy early on. At 6 months of age, infants are able to perceive the difference between small sets of elements varying in quantity (2 vs. 3-object sets), and can even distinguish between larger quantities, provided that the ratio between two sets is large enough (ex. 16 vs. 32, but not 8 vs. 12). These preverbal representations become more refined over time, and they form the early, though not sufficient, building blocks of future mathematical learning.

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One achievement in numeracy is the acquisition of fact fluency. Fact fluency refers to the knowledge necessary to produce sums and differences in a flexible, timely and accurate manner. In the toddler years, children progressively acquire the requirements for fact fluency, often beginning with intuitive numbers (ex. know the meaning of one, two, three), leading to the ability to recognize that, for example, any set of three elements has a larger count than a set of two elements.

As they get older, children develop more advanced number skills. By age 3, they begin to be proficient in some nonverbal, object-based tasks, such as understanding the process of adding and subtracting, and judging one set as having a larger quantity than a second one. Although preschoolers can match collections of 2, 3, and 4 elements if the objects are of similar size or shape, they still struggle when the objects are highly dissimilar (ex. matching two animal figurines with two black dots). Preschool children are also likely to get easily distracted by superficial features of a set (ex. judging a set of items as having a larger quantity than another equal set because the items are disposed in a longer row). The development of number sense depends on level of representation and set size and is critical for setting mathematics trajectories in mathematics throughout elementary school. Research is currently under way to determine how knowledge about quantities in infancy is related to preschool numerical competencies and later school achievement.

Although most children can naturally discover mathematical concepts, environment and cultural experiences play a role in advancing children’s knowledge about numbers. For instance, language acquisition allows children to solve verbal problems and develop a number sense (ex., understanding cardinality, the total number of elements in a set). Children who lack early experiences with numbers tend to lag behind their peers. For instance, children from economically disadvantaged families tend to display poor numeracy skills early on, and these deficiencies later translate to mathematical difficulties in school. Performance on numerical problems and the kinds of cognitive strategies children use are likely to vary considerably across children. Even the range of one child’s responses from one trial to the next can be substantial.

The process of building the number sense is gradual and begins in the preschool years. Promoting early competencies in numeracy is important because of its relation to children’s mathematical readiness at school entry and beyond. Preschool children who have acquired the ability to count, name numbers, and make distinctions between different quantities tend to

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perform well on numerical tasks in kindergarten. In addition, children's good numerical abilities predict later school achievement more strongly than their reading, concentration, and socioemotional skills.

What can be done?

Given children's natural dispositions to learn about numbers, they should be encouraged to freely explore and practice their abilities in a range of unstructured activities. These learning experiences should be enjoyable and developmentally appropriate so that children stay engaged in the activity and do not get discouraged. Playing board games and other activities involving experiments with numbers can help children develop their numeracy skills. Materials such as blocks, puzzles, and shapes can also encourage the development of numeracy.

Parents can foster their child's numerical knowledge by creating meaningful experiences with numbers paired with appropriate feedback (ex. asking the child how many feet she has, and using her response to explain why she needs two, and not one shoe). Parents and teachers should also create spontaneous educational moments that encourage the child to think and talk about numbers. Numbers can be introduced in several domains, including play (dice-throwing games), art (drawing a number of stars), and music (keeping a tempo of 2 or 3 beats).

Taking on children's perspective and understanding that their interpretations of mathematical problems are different than adults' are important components of effective education. Learning trajectories can facilitate engaging and developmentally appropriate teaching and learning for all children. Teachers need to know that numeracy follows a developmental process, and numerical activities must therefore be designed accordingly. To optimize interventions aimed at numeracy, kindergarten screening should ensure that children can recognize the quantity of small sets of objects (2 and 3) and make the distinction between these and larger sets (4 or 5 objects).

Early interventions in mathematics have important implications for school readiness. A successful ECME program includes a stimulating environment containing objects and toys that encourage mathematical reasoning (ex., table blocks and puzzles), play opportunities where children can develop and expand their natural mathematical abilities on their own, and teachable moments where preschool teachers ask questions about children's mathematical discoveries.

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Numerical Knowledge in Early Childhood

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Introduction

Research on the numerical knowledge of young children has grown rapidly in recent years. This research encompasses a wide range of abilities and concepts, from infants' ability to discriminate between collections containing different numbers of elements[1,2] to preschoolers' understanding of number words[3,4] and counting,[5,6,7] and their grasp of the inverse relation between addition and subtraction.[8,9]

Subject

Research on young children's numerical knowledge provides an important foundation for the formulation of standards for early childhood education[10] and for the design of early childhood mathematics curricula.[11,12,13] Further, the mathematics knowledge that children acquire before they begin formal schooling has important ramifications for school performance and future career options.[14] An analysis of predictors of academic achievement, based on six longitudinal data sets, found that children's math skills at school entry predicted subsequent school performance more strongly than did early reading skills, attentional skills or socioemotional skills.[15]

Problems

Fundamentally, numeracy entails understanding numbers as representations of a particular kind of magnitude. Correspondingly, understanding the development of numeracy in early childhood entails understanding both how children come to understand the basic quantitative relations that numbers share with other kinds of quantities and how they come to understand the aspects of number that distinguish it from other kinds of quantities.

Research Context

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Piaget's classic research on logico-mathematical development investigated children's understanding of general properties of quantity such as seriation and the conservation of equivalence relations under certain kinds of transformations.[16] His view, however, was that this kind of knowledge emerges only with the acquisition of concrete-operational thinking, around 5-7 years of age. Subsequent researchers[17] undertook to demonstrate that younger children have considerably more numerical knowledge than Piaget recognized; and contemporary research provides evidence of a wide range of early numerical abilities.[18]

Key Research Questions

An influential but controversial claim in current research literature on early numerical abilities holds that the brain is "hard wired" for number.[19,20] This idea is often supported by evidence of numerical discrimination by human infants and by animals.[21] Critics of innatist (philosophical doctrine that holds that the mind is born with ideas/knowledge) accounts of numerical knowledge, however, note the pervasiveness of developmental change in numerical reasoning,[22] the slow differentiation of number from other quantitative dimensions,[23] and the contextualized nature of early numerical knowledge.[24] Further, accumulating evidence indicates that language[24] and other cultural products and practices[25,26] make enormous contributions to young children's acquisition of numerical knowledge.

Recent Research Results

Numerical knowledge in infancy

One of the most active areas of current research concerns the numerical abilities of infants. Kobayashi, Hiraki and Hasegawa[1] used discrepancies between visual and auditory information about the number of items in a collection to test for numerical discrimination in six-month-olds. They showed infants objects that made a sound when dropped onto a surface, and then dropped two or three of the objects behind a screen so that the infants heard the tone each item made but could not see the items. They then removed the screen to reveal either the correct number of objects or a different number (3 if there had been 2 tones, and vice versa). Infants looked longer when the number of items revealed did not match the number of tones, indicating that they were able to distinguish between two and three items. Other research indicates that six-month-old infants can also discriminate between larger numerical quantities, provided the numerical ratio between them is large. Six-month-old infants discriminate between 4 vs.8[27] and even 16 vs. 32.[28]

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When the contrast is reduced (for example, 8 vs. 12), however, six-month-old infants fail[29] but older ones succeed.[2] Thus, infants become able to make finer numerical discriminations as they get older.

Young children's knowledge about numerical relations

Because numbers represent a kind of magnitude, a fundamental aspect of numerical knowledge pertains to equal, less-than and greater-than relations between numerical quantities.[30] Somewhat surprisingly, in light of the infancy findings, it is a significant developmental achievement for preschool children to compare sets numerically, particularly when that entails disregarding other differences between the sets.

For example, Mix[31] studied the ability of three-year-olds to numerically match a set of 2, 3 or 4 black dots. This task was easy when the manipulatives children were given were perceptually similar to the dots they were to match (e.g., black disks, or red shells about the same size as the dots). However, children's performance dropped when the manipulatives contrasted perceptually with the dots (e.g., lion figurines or heterogeneous objects).

Muldoon, Lewis, and Francis[7] assessed four-year-olds' ability to evaluate the numerical relation between two rows of blocks (with 6-9 items per row) in the face of misleading length cues, that is, when two unequal-length rows contained the same number of items, or two equal-length rows contained different numbers of items. Most children relied on length comparisons rather than on counting the items to compare the rows. However, a three-session training procedure led to better performance, particularly among children who, as part of the training, were asked to explain why the rows were in fact numerically equal or unequal (as indicated by the experimenter).

Research Gaps

While experimental data concerning early numeracy is accumulating rapidly, the absence of theoretical accounts that incorporate the full range of empirical results limits our understanding of how the diverse findings already obtained fit together and what issues remain to be resolved. In the infancy literature, for example, competing accounts of early numerical abilities have generated much research in the past few years, yet the findings have not lessened the theoretical controversy. In advancing theoretical conclusions, researchers need to be cognizant of the entire corpus of findings, and their theories need to be formulated precisely enough that they can be

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differentiated empirically.

In addition, researchers need to gather better information about the processes that lead to advances in early numeracy knowledge. We know that young children's performance is affected by contextual variables ranging from culture and social class[32] to patterns of parent-child[33,34] and teacher-child[35] interaction. As yet, however, we have only small pieces of information, mostly from experimental training studies[7,25,36] about how particular experiences alter children's numerical thinking. Research that provides converging data about (a) young children's everyday numerical experiences, and how they vary with the age of the child, and (b) the experimental effects of those kinds of experiences on children's thinking, would be especially helpful.

Conclusions

The available research on young children's developing knowledge about number supports four generalizations that have important implications for policy and practice. First, numerical development is multifaceted. Early childhood numeracy encompasses much more than counting and knowing some elementary arithmetic facts. Second, notwithstanding the number-related abilities evidenced even by infants, age-related change is pervasive. In age group comparisons, the older children nearly always perform better. Third, variability is pervasive. Individual children vary in their performance across different numerical tasks,[37] in their engagement in particular sorts of numerical reasoning across different contexts,[3] and even in their trial-to-trial responses within a single task.[5,38] Finally, children's progress in acquiring numerical knowledge is highly malleable. It is influenced by informal activities such as playing board games,[25] by experimental activities designed to illuminate numerical relationships,[7,36] and by variations in the ways in which parents[33,34] and teachers[35] talk to children about numbers.

Implications

An important contribution that research on early childhood numeracy can make to policy and practice is to inform the goals we set for early mathematics instruction. Just as numerical development in early childhood is multi-faceted, the goals of early childhood instructional programs should be much broader than enhancing children's counting skills or teaching them some basic arithmetic facts. Numbers, like other kinds of magnitudes, are characterized by relations of equality and inequality. At the same time, they differ from other kinds of magnitudes in that they are based on the partitioning of an overall quantity into units. Instructional activities

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that encourage children to think about relationships between quantities and effects of transformations such as partitioning, grouping, or rearranging those relationships may be helpful in advancing children's understanding of these ideas. The variability and malleability of young children's numerical thinking indicate the potential for early childhood instructional programs to contribute substantially to children's growing knowledge about numbers.

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Early Predictors of Mathematics Achievement and Mathematics Learning Difficulties

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Introduction

Mathematics difficulties are widespread. Up to 10% of students are diagnosed with a learning disability in mathematics at some point in their school careers.[1,2] Many more learners struggle in mathematics without a formal diagnosis. Mathematics difficulties are persistent, and students who have difficulties may never catch up to their normally achieving peers without intervention.

Subject

Foundations for mathematics achievement are established before children enter primary school.[3,4] Identification of key predictors of mathematics outcomes provides support for screening, intervention and progress monitoring before children fall seriously behind in school.

Problem

The consequences of poor mathematics achievement are serious for everyday functioning, educational attainment, and career advancement.[5] Mathematics competence is necessary for entry into STEM (science, technology, engineering and mathematics) disciplines in college and for STEM-related occupations.[6] There are large group differences in mathematics achievement related to socioeconomic status[7] as well as individual differences in general learning abilities.[8] These disparities are already present in early childhood and increase over the course of schooling.

Research Context

Longitudinal studies of characteristics of children with mathematics difficulties have identified important targets for intervention. Most children enter school with *number sense* that is relevant to learning school mathematics. Preverbal components of number (e.g., perceiving exact representations of small sets of objects and approximate representations of larger sets) develop

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in infancy.[9,10,11,12] Although these primary foundations are thought to underlie learning of conventional mathematics skills, they are not sufficient. Most children with difficulties in mathematics show weaknesses in number sense related to knowledge of number, number relations, and number operations[4,13] – strands of number sense are malleable and influenced by experience.[14] Early number relates to knowledge of oral and written number and counting concepts, such as one-to-one correspondence and cardinality. Number relations involve understanding of numerical magnitudes on the number line. Number operations relates to transforming quantities through addition and subtraction.[4,15]

Key Research Questions

Early competencies that are aligned with the mathematics children are required to do in school are most predictive of mathematics achievement and difficulties.[16] Assessment tools and interventions need to be refined to help children develop key number sense concepts. Identifying pathways and factors related to number sense development is needed to guide the creation of early childhood interventions for those at-risk for developing mathematics learning difficulties in school.

Recent Research Results

Early number sense sets children's achievement trajectories in mathematics.[16,17,18,19,20] Mathematics difficulties and disabilities have their roots in poorly developed number sense.[21,22,23] Children with developmental dyscalculia, a severe form of mathematics disability, are characterized by deficits in counting and enumerating sets of objects and in recognizing and comparing numbers.[21] Such deficits lead to poor arithmetic fluency, a hallmark skill in the primary grades.

Number sense as a predictor of later mathematics achievement and difficulty

Studies provide empirical evidence of a multifactor early number sense model consisting of specific strands of number, number relations, and number operations understanding.[24,25] Early screening tools developed using this model accurately identify children at risk for mathematics learning difficulty and disability.[26,27,28] Predictive relations may differ by level of number sense in preschool. Number strand skills predict later mathematics achievement for children with low and intermediate achievement, but not high achievement; number relations strand skills predict later achievement for children of all math achievement levels; number operations strand skills predict

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later math achievement for children with intermediate and high achievement, but not low achievement.[29]

Low-income children enter kindergarten well behind their middle-income peers on most symbolic numeracy indicators, and this gap does not narrow during the school year.[13] Longitudinal studies over multiple time points, from the beginning of kindergarten through the end of Grade 3, suggest that foundational number sense supports the learning of complex mathematics associated with computation as well as applied problem solving and fluency.[16,20,30,31] The low mathematics achievement of high-risk, low-income students is mediated by kindergarten number sense. Because early number competencies are achievable in most children[4] their intermediate effects provide clear directions for early intervention

Type of quantity representation and set size

The size of a quantity or set along with the way it is presented to a child (e.g., non-symbolic or symbolic) affects children's reasoning about numbers. Children's ability to map written numerals to the quantities they represent is critical for learning more complex number sense skills.[32] Non-symbolic quantity representations (e.g., which of 2 sets of dots has more, without counting) scaffold the development of symbolic understanding (which of 2 numerals is bigger), but only for small sets (i.e., 4 or less).[33] The results suggest that children may be able to engage in both symbolic and non-symbolic number sense activities across strands (number, number relations, and number operations) with small set sizes. An intervention in which children engaged in a variety of number sense skills with small set sizes before cycling to a similar sequence with larger set sizes was successful in kindergartners at-risk for later mathematics learning difficulties.[34]

Developmental pathways

Research has revealed patterns of individual differences in early number sense development. There are empirically distinct developmental pathways in number, number relations, and number operations for preschoolers across the school year,[35,36] which predict mathematics achievement in Grades 1 and 3.[35] Low receptive vocabulary[35,36] and visual-spatial working memory skills[34] predict membership in a consistently low developmental pathway, emphasizing the importance of domain-general learning skills for early numeracy development.[37] Contextual factors such as children's home learning environment also relate to individual differences in early numerical

development.[38]

Research Gaps

Additional work is needed to consider how the number sense strands of number, number relations, and number operations work together during the early childhood period. Researchers must also consider how set size and level of representation constrain the development of number sense, including for children at-risk for mathematics learning difficulties. Interventions that target and weave together the strands of number sense for children with or at-risk for mathematics learning difficulties should be developed and evaluated through randomized-controlled studies.

Conclusions

Difficulties with mathematics are pervasive and can have lifelong consequences. Foundational number sense skills develop in early childhood and are highly predictive of mathematics achievement and difficulties. The development of number sense depends on level of representation and set size. Research suggests that number sense should be prioritized in preschool and kindergarten to provide a foundation for learning formal arithmetic and developing fluency. Overall, early number sense is critical for setting mathematics trajectories in mathematics throughout elementary school.

Implications for Parents, Service, and Policy

In contemporary educational settings, challenges in learning mathematics may go unnoticed until Grade 4. Early interventions in mathematics are less common than are those for reading, although early screening and multi-tiered intervention programs are growing as we expand our knowledge. Preschools and kindergartens should incorporate mathematics experiences that emphasize instruction in number, number relations and number operations. The curriculum should gradually increase set size and vary type of representation.4,34 It is crucial for curriculum developers in early childhood education to concentrate on fundamental aspects of number sense. By doing so, early interventions can equip all children with the necessary foundations for success in formal mathematics.

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Early Numeracy: The Transition from Infancy to Early Childhood

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Introduction

By preschool age, most children exhibit a range of numeracy skills, including verbal skills, such as counting, and nonverbal skills, such as recognizing equivalence of object sets.[1,2,3,4,5] Although researchers agree that these abilities are present in early childhood, they continue to debate when, and by what mechanisms, these abilities emerge. In other words, what are the developmental origins of verbal and nonverbal numerical competencies?

Subject

Research on numeracy traditionally focused on verbal counting. However, the notion that numeracy might emerge in infancy and toddlerhood shifted the focus toward concepts that can be measured nonverbally. This shift expanded the range of behaviours included in early numeracy—a change that has direct implications for early childhood education and assessment. This shift also raised questions about the developmental origins of mathematics learning difficulties and gaps in mathematical achievement.

Problems

Most children acquire basic symbolic number skills by 5 years of age, such as reciting the count list to 20 or more,[1,2,4] using the count list to enumerate various sets,[1,2,4,5] understanding that the last word in a count stands for the numerosity of the set (i.e., Cardinal Word Principle or CWP),[1,2,5,6] identifying written numerals,[5] and judging ordinality of single-digit numerals.[7] There is also emerging evidence that children can interpret multidigit numbers starting at 3 years of age—correctly judging, for example, which of two multidigit numerals is larger.[8,9]

Prior to mastery of symbolic skills, preschool children also exhibit understanding of quantitative relations on nonverbal measures, such as matching equivalent sets of objects,[10] performing

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simple calculations with objects,[11] or indicating which of two dot clouds has more.[12] Children perform object-based number tasks earlier than they demonstrate similar understandings in verbal tasks. For example, preschoolers solve simple object-based addition and subtraction problems (e.g., 2 + 2) years before they can solve analogous verbal problems.[11,13] Similarly, children judge ordinality and equivalence in forced choice tasks earlier than they can compare the same sets verbally, via counting, with nonverbal competence emerging between 2-1/2 and 3 years of age.[11,14,15]

A major research focus has been understanding the developmental origins of these nonverbal number concepts. Researchers have shown, using habituation and preferential looking methods, that infants are sensitive to quantity as well,[16,17] with some studies demonstrating this sensitivity in newborns.[18,19] Various proposals have linked individual variation in this early sensitivity to later numeracy and mathematics outcomes. However, open questions remain about the representations underlying this early sensitivity, how the representations themselves develop, and what role these representations may play in subsequent development.

Research Context

One candidate for early nonverbal number representation is the Approximate Number System, or ANS—a representation proposed to underlie the discrimination of different set sizes—particularly large set sizes (e.g., 16 vs. 32).[20,21] Although the ANS is thought to operate over discrete number, it is also inexact and ratio-dependent, similar to non-numerical dimensions such as surface area, meaning that quantities are easier to tell apart when their ratio is higher (e.g., 16 is easier to discriminate from 32 than from 24).[22] The ANS is considered innately available because even newborns respond to variations in set sizes as long as the ratios are large enough.[19] However, research also shows that with age and schooling, the ANS becomes more precise.[23,24,25,26]

Another proposed nonverbal number representation is based on object individuation, also described as object tracking, mental models, or subitizing—the immediate perception of number in small quantities (e.g., 1 to 4 objects).[11,21,27,28] On these accounts, children incidentally represent number when they differentiate objects in a scene and keep track of the objects' movements and spatial positions. Set size limits on object individuation have been explained by constraints on working memory[29] or attention.[27] Some have argued that like the ANS, object-based representations are an innate endowment, with ongoing debate about whether the two systems are distinct[11,30] or simply different instantiations of the same evolutionarily primitive

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representational system.[31] Still others have suggested object-based representations could emerge from experiences observing and manipulating objects without necessarily arising from an innate quantification system.[32]

A third contributor to early numeracy is exposure to number words and the verbal counting system. Prior to the advent of research on infant quantification, seminal research by Piaget suggested that children lacked a conceptual understanding of quantitative relations until well after they had mastered conventional counting[33] and studies showed that children did not understand numeracy principles until after they had mastered counting procedures.[34,35] Although subsequent research has shown that precounting children understand much more about quantities than Piaget claimed, symbolic number understanding remains a strong predictor of later mathematics achievement,[36,37,38,39,40] and indeed, stronger than nonverbal quantification skills.[41,42,43,44] Research has also suggested children can extract information about numbers and their meanings from numeric symbols themselves, showing for example, that preschool children can match written multidigit numerals to multidigit number words and compare magnitudes of written multidigit numerals independent of performance using nonverbal measures.[9]

Key Research Questions

Most researchers agree that children respond to changes in number early in life via nonverbal processes. Furthermore, there is general agreement about the stages of verbal number acquisition. Current research is now focused on the underlying nature of nonverbal quantification and whether variation in nonverbal processes is related to later mathematics achievement. In this research, investigators also consider whether children bootstrap between verbal and nonverbal quantification as they learn.[45] Finally, there is growing interest in the verbal numeracy environment at home and in preschool, and its connection to later child outcomes.

Recent Research Results

Early number discrimination and non-numerical quantitative dimensions

There is ongoing debate about whether infants' responses to quantitative changes are based on an awareness of discrete number *per se*, or one of many perceptual variables that correlate with discrete number, such as surface area, convex hull, brightness, duration, temporal density, and spatial frequency.[46,47,48] Researchers have attempted to control these perceptual variables to

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obtain a clean test of numerical sensitivity,[24,49,50] but it is difficult to control all of these perceptual variables simultaneously, as others have pointed out,[46,47] leading some to suggest that future research should focus on ways to account for non-numerical responses rather than attempting to control them.[46,50,51] Thus, it remains unclear whether infants' quantitative sensitivity is based on discrete number, as some have claimed, or a combination of other perceptual information that is correlated with discrete number. Similar issues arise in research testing whether infants respond to changes in quantity across dimensions—for example, learning to associate certain visual patterns with larger and smaller numerical sets and transferring this association to objects differing in size,[52] or expecting that if quantitative pairs (e.g., number and spatial extent) both increase or decrease, they will both change in the same direction[18] —research which has led to the proposal that quantification arises from a generalized magnitude representation. Such a representation is one way to characterize an undifferentiated sense of quantity based on multiple input streams, but the claim that children can switch from one quantitative cue to another would require controls that can isolate each cue effectively.

Making connections

Research has documented how children acquire several distinct verbal enumeration skills (e.g., counting, cardinality, ordinality), as well as how they represent quantities nonverbally. However, to achieve a coherent number concept, children must eventually make connections among these skills and representations (e.g., verbal number words, physical quantities, mental models).[43,53,54,55,56,57] Small number words may play a critical role in children's first mappings because the quantities one, two, and three can be immediately perceived and represented nonverbally with less error than representations of larger quantities. Thus, small sets may offer clear perceptual referents that can be labeled with a number word.[28,58,59,60]

Once the labels for small sets have been learned, children are positioned to notice that the same words are used as labels and in counting, thereby discovering the Cardinal Word Principle (CWP)—the idea that the last word in a count stands for its cardinal number. In the absence of targeted instruction, most children naturally attain the CWP by age 4 years, but studies have shown the CWP can be induced from practice labeling small sets as well as instruction that juxtaposes counting and labeling.[28,61] In the *n*-knower framework, CWP has been measured using the *Give-n* task (e.g., "Give me 5 counters."), and early research findings suggested children learn number-to-quantity mappings one by one and in order, prior to making the connection between counting and cardinality, which itself is followed by a rapid logical generalization of the

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CWP to all the numbers within a child's counting range.[45,62,5] However, diary studies have reported correct use of small number words in certain contexts even earlier, as well as evidence that children may acquire these number meanings in a different order.[63,64] Moreover, recent studies have raised questions about the validity of *Give-n* performance and the meaning of *n*-knower classifications based on it.[65,66,67,68] Thus, although much has been learned about these important connections, key questions remain unresolved.

Early predictors of mathematical achievement

Evidence of quantitative sensitivity in infancy has inspired researchers to examine how this sensitivity relates to acquisition of verbal numeracy in early childhood, as well as eventual mathematical achievement in school. Some have argued that the ANS provides a representational foundation for acquisition of later symbolic numeracy and mathematics skills[21,45] and longitudinal studies linking ANS acuity in infancy and preschool to later mathematical achievement in childhood and adolescence seem to bolster this claim.[69,70,71,72] However, other studies examining longitudinal and concurrent associations have failed to find evidence connecting ANS acuity to mathematics achievement,[73,74,75,76,77] and indeed, accumulating neurological and behavioural evidence points to separate mechanisms.[12,26,78] Finally, when children acquire symbolic numeracy skills, ANS acuity improves concurrently, perhaps as a result.[25,79] Thus, if ANS and symbolic mathematics skills are causally related, the relation could be from symbolic number to ANS rather than the reverse, or perhaps, bidirectional.

Similar patterns have been reported for spontaneous focusing on number (SFON)—the tendency of children to notice exact number in their daily experiences.[80] Tests of SFON carefully avoid verbal number cues in order to tap children's self-directed attention to numerosity, but because children in these studies are generally preschool aged or older,[80,81] it is unclear whether the mechanism driving SFON is nonverbal quantification (e.g., object individuation), verbal counting, or both. Concurrent correlational studies indicate strong associations between SFON tendencies and verbal numeracy,[80,82] and longitudinal studies demonstrate that performance on SFON measures in early childhood is correlated with symbolic number knowledge in elementary school;[83,84] however, whereas both SFON and symbolic numeracy predict subsequent mathematics achievement, performance on symbolic number tasks is the stronger predictor.[85] Also, attempts to improve SFON have been successful when interventions included symbolic number activities,[86,87] suggesting that SFON itself may be driven by symbolic numeracy acquisition, rather than the reverse. Additional research using interventions based on nonverbal activities is needed to draw

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firm conclusions, but early symbolic numeracy remains the clearest and most potent predictor of later mathematics achievement.

Home Numeracy Environment

Acquisition of children's first numeracy skills takes place largely in the family home, so the number-related activities of children and their caregivers have received increasing attention.[88,89] Most research on this topic has used either parent report of numeracy activities[90,91,92,93,94] or coding parent speech from direct observations.[95,96,97] Studies have demonstrated an association between the frequency of home numeracy activities based on parent report and children's numeracy outcomes,[90,91] though this association is not always obtained.[89,92,93] Existing studies also indicate that although parents talk about number infrequently,[95] even when activities are designed to elicit such talk,[60,97] there are significant associations between the frequency of incidental number talk and children's numeracy outcomes.[90,95,96,97,98] Child outcomes have also been linked to variation in qualitative differences, such as conversation length,[99] and focusing on large set sizes (e.g., 4-10) or advanced concepts such as cardinality.[97,100,101] A few observational studies have targeted infancy in particular, demonstrating that parental number talk is present at the youngest ages observed to date (i.e., 12 to 14 months).[95,96] Thus, although infants are themselves nonverbal, their emerging understandings of numeracy may be shaped by exposure to verbal numeracy early on.[32]

Research Gaps

Although research has generated extensive information about developmental changes in various quantitative skills, such as the CWP, SFON, and nonverbal set size discriminations, less is known about the mechanisms that drive these changes, and particularly, the mechanisms by which children make connections among various concepts and representations to achieve a coherent sense of number. Related to this issue, more research is needed to test proposed mechanisms experimentally, by providing inputs that are consistent with hypotheses about developmental mechanism. For example, though it has been argued that small set sizes offer an opportunity to unite verbal and nonverbal quantification, the next step is to demonstrate that this is the case experimentally. Intervention studies that test the effects of specific input types may also be helpful in this regard. Similarly, more research examining the relations between verbal number and nonverbal number are needed to determine what directions of influence are at play, at what ages, and under what conditions.

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Another persistent issue that remains unresolved is whether nonverbal quantification is based on discrete number or attention to non-numerical variables, such as surface area. Although researchers have focused on attempts to control these non-numerical variables, a promising alternative may be to design measures that account for non-numerical responses rather than attempting to control for them.[46,50,51]

Finally, intriguing new research about the home numeracy environment as well as the origins of multidigit number concepts have raised a host of new questions that bear investigation. For example, most studies of children's home numeracy environment have focused on preschool age, but much could be gained by tracing these experiences back into infancy, particularly given the longstanding evidence of nonverbal quantification in this age range. Are infants directed to attend to number much earlier in life than we have documented to date? If so, how might this change our understanding of SFON, for example. Similarly, the unexpectedly early acquisition of multidigit number meanings raises new questions about the presence of multidigit numeracy in parents' number talk, as well as whether variation in these informal insights is related to subsequent mathematics outcomes. Interventions targeting either the home numeracy environment, early multidigit numeracy, or both, would be exciting new directions for future research.

Conclusions

Evidence of numerical competence in infants has raised intriguing questions about the origins of numeracy and the conceptual resources young children use to acquire verbal counting. However, further research is needed to reveal precisely how this infant competence connects to subsequent nonverbal and verbal development and whether these mechanisms can be leveraged to help all children enter school with a strong foundation of numeracy concepts.

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Learning Trajectories in Early Mathematics – Sequences of Acquisition and Teaching

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Introduction

Children follow natural developmental progressions in learning and development. For example, children first learn to crawl, then walk, run, skip, and jump with increasing speed and dexterity. Similarly, they follow natural developmental progressions in learning math; they learn mathematical ideas and skills in their own way.[1] When educators understand these developmental progressions, and sequence activities based on them, they can build mathematically enriched learning environments that are developmentally appropriate and effective. These developmental paths are a main component of a *learning trajectory*.

Key Research Questions

Learning trajectories help us answer several questions.

1. What goals should we establish?
2. Where do we start, that is, what are children's developmental level?
3. How do we know where to go next?
4. How do we get there?

Recent Research Results

Recently, researchers have come to a basic agreement on the nature of learning trajectories.2 Learning trajectories have three parts: a) a mathematical goal; b) a developmental path along which children develop to reach that goal; and c) a set of instructional activities, matched to each of the levels of thinking in that path, that help children develop higher levels of thinking. Let's examine each of these three parts.

Goals: The Big Ideas of Mathematics

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The first part of a learning trajectory is a *mathematical goal*. Goals involve the *big ideas of mathematics*—clusters of concepts and skills that are mathematically central and coherent, consistent with children's thinking, and generative of future learning. These big ideas come from several national efforts.[3-6] For example, one big idea is that *counting can be used to find out how many are in a collection. Another would be, geometric shapes can be described, analyzed, transformed and composed and decomposed into other shapes.* It is important to realize that there are several such big ideas and learning trajectories.

Development Progressions: The Paths of Learning

The second part of a learning trajectory consists of levels of thinking; each more sophisticated than the last, through which most children progress on their way to achieving the mathematical goal. That is, the developmental progression describes a typical path children follow in developing understanding and skill about that mathematical topic. Development of mathematics abilities begins when life begins. Young children have certain mathematical-like competencies in number, spatial sense, and patterns from birth.[1,4]

However, young children's ideas and their interpretations of situations are uniquely different from those of adults. For this reason, early childhood teachers are careful not to assume that children "see" situations, problems, or solutions as they do. Instead, teachers interpret what the child is doing and thinking, they attempt to see the situation from the child's point of view. Similarly, when these teachers interact with the child, they also consider the instructional activities and their own actions through the child's eyes. This makes early childhood teaching both demanding and rewarding.

Learning trajectories provide simple labels and descriptions for each level of thinking in every mathematical topic. Figure 1 illustrates a part of the learning trajectory for counting. The Developmental Progression column provides both a label and description for each level. It is important to note that the ages in the first column are approximate. Without experience, some children can be years behind this average age. With high-quality education, children can far exceed these averages. [For complete learning trajectories for all topics, including the research on which they are based, see references[1,7,] as well as LearningTrajectorie.org].

Instructional Activities: The Paths of Teaching

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The third part of a learning trajectory consists of set of instructional strategies and activities, matched to each of the levels of thinking in the developmental progression, designed to help children learn the ideas and skills needed to achieve that level of thinking. That is, as teachers, we can use these strategies and activities to promote children's growth from one level to the next. The third column in Figure 1 provides examples.

Table 1. Sample Levels from the Learning Trajectory for Counting [From references[1,7,] as well as LearningTrajectorie.org]

Age	**Developmental Progression**	**Instructional Activities**
1 year	**Number Word Sayer: Foundations.** No verbal counting but names some number words. **Chanter** Chants in singsong fashion or sometimes-indistinguishable number words.	Associate number words with small quantities (see “Subitizing” in the resources) and verbally count for fun (e.g., going up stairs).
2	**Reciter** Verbally counts with separate words, not necessarily in the correct order.	Provide repeated, frequent experience with the counting sequence in varied contexts. *Count and Race.* Children verbally count along with the computer (up to 50) by adding cars to a racetrack one at a time.
3	**Reciter (10)** Verbally counts to ten, with *some* correspondence with objects.	*Count and Move.* Have all children count from 1-10 or an appropriate number, making motions with each count. For example, say, “one” [touch head], “two” [touch shoulders], “three” [touch head], and so forth.
	Corresponder Keeps one-to-one correspondence between counting words and objects (one word for each object), at least for small groups of objects laid in a line.	*Counting Wand.* Children use a counting wand to count the number of children in a group, focusing on the 1-to-1 correspondence.

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Age	Developmental Progression	Instructional Activities
4	**Counter (Small Numbers)** Accurately counts objects in a line to 5 and answers the “how many” question with the last number counted.	*Cubes in the Box.* Have the child count a small set of cubes. Put them in the box and close the lid. Then ask the child how many cubes you are hiding. If the child is ready, have him/her write the numeral. Dump them out and count together to check. *Pizza Pizzazz 2* Children count items up to 5, putting toppings on a pizza to match a target amount.
	Producer —Counter To (Small Numbers) Counts out objects to 5. Recognizes that counting is relevant to situations in which a certain number must be placed.	*Count Motions.* While waiting during transitions, have children count how many times you jump or clap, or some other motion. Then have them do those motions the same number of times. Initially, count the actions with children.
5	**Counter and Producer (10+)** Counts and counts out objects accurately to 10, then beyond (to about 30). Has explicit understanding of cardinality (how numbers tell how many). Keeps track of objects that have and have not been counted, even in different arrangements.	*Counting Towers (Beyond 10).* To allow children to count to 20 and beyond, have them make towers with other objects such as coins. Children build a tower as high as they can, placing more coins, but not straightening coins already in the tower. The goal is to estimate and then count to find out how many coins are in your tallest tower.

In summary, learning trajectories describe the goals of learning, the thinking and learning processes of children at various levels, and the learning activities in which they might engage. People often have several questions about learning trajectories.

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How Do Learning Trajectories' Developmental Levels Support Teaching and Learning? The levels help teacher understand children's thinking; create, modify, or sequence activities. Teachers who understand learning trajectories are more effective and efficient and engage children in mathematics joyfully. Through planned teaching and also by encouraging informal, incidental math, teachers help children learn at an appropriate and deep level.

There are Ages in the Learning Trajectories. Should I Plan to Help Children Develop Just the Levels that Correspond to my Children's Ages? The ages in the table are typical ages at which children develop these ideas. But these are rough guides only—children differ widely. Furthermore, the children achieve much later levels with high-quality education. So, these are approximate levels to help orient educators not goals. Children who are provided high-quality math experiences are capable of developing to levels one or more years beyond their peers.

Are the Instructional Activities the Only Way to Teach Children to Achieve Higher Levels of Thinking? No, there are many ways. In some cases, however, there is some research evidence that these are especially effective ways. In other cases, they are simply illustrations of the kind of activity that would be appropriate to reach that level of thinking. Further, teachers need to use a variety of pedagogical strategies in teaching the content, presenting the activities, guiding children in completing them, and so forth.

Future Directions

Although learning trajectories have proven to be effective for early mathematics curricula and professional development8-10, much remains to be studied, such as learning trajectories for older students. Also, in the early years, several learning trajectories are based on considerable research, such as those for counting and arithmetic. However, others, such as patterning have a smaller research base. These remain challenges to the field.

Conclusions

Learning trajectories hold promise for improving professional development and teaching in the area of early mathematics.8,11,12 Further, researchers suggest that professional development focused on learning trajectories increases not only teachers' professional knowledge but also their students' motivation and achievement.9,13-15 Thus, learning trajectories can facilitate engaging and developmentally appropriate teaching and learning for all children.

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Fostering Early Numeracy in Preschool and Kindergarten

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Introduction

Over the last six decades, it has become increasingly clear that children's everyday (informal) mathematical knowledge is an important basis for learning school (formal) mathematics.[1,2,3] Consider the long debated issue: How can students best be helped to achieve fluency with the single-digit (basic) addition facts, such as 3+4=7 and 9+5=14, and related subtraction facts, such as 7–3=4 and 14–9=5 (see, e.g., Baroody & Dowker,[4] particularly chapters 2, 3, 6, and 7)? (Fact fluency entails generating sums and differences quickly and accurately and applying this knowledge **appropriately** and **flexibly**[5].) Research indicates that helping children build number sense in the preschool and primary-grade years can promote fact fluency.[6,7,8,9] The aim of this entry is to summarize how the development of informal number sense before kindergarten and grades K and 1 provides a foundation for the key formal skill of addition and subtraction fact fluency in grades 2 and 3.

Key Research Questions

1. When should parents and early childhood educators begin (a) the process of promoting number sense and (b) efforts to foster fact fluency directly?
2. What are the developmental prerequisites preschoolers and kindergartners need to achieve fluency with basic sums and differences efficiently and effectively?
3. What role does language play in the development of this foundational knowledge?
4. How can parents and early childhood educators most effectively encourage number sense and fact fluency?

Recent Research Results

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Question 1. The process of helping children build number sense—the foundation of fact fluency—can and should begin in the preschool years. Recent research indicates that children begin to construct number sense very early. Indeed, some toddlers as young as 18 months and nearly all 2-year-olds have begun learning the developmental prerequisites for fact fluency (e.g., see Baroody, Lai, & Mix,[1] for a review).

Successful efforts to promote fact fluency depend on ensuring a child is developmentally ready and on not rushing the child. As research indicates significant individual differences in number sense appear as early as two or three years of age and often increase with age,[1,10] there are no hard and fast rules about when formal training on fact fluency should begin. For many children though, this training, even with the easiest (n+0 and n+1) sums, may not be developmentally appropriate until late kindergarten or early first grade.[11] For children at risk for academic failure, work with even the easiest sums often does not make sense until first or second grade.[12]

Questions 2 and 3. Some research indicates that language, in the form of the first few number words, plays a key role in the construction of number sense (for a detailed discussion, see Baroody;[3] Mix, Sandhofer, & Baroody[13]). More specifically, it can provide a basis for two foundations of early number sense—namely a concept of cardinal number (the total number of items in a collection) and the skill of verbal number recognition (VNR), usually called "subitizing," shown at the apex of Figure 1. VNR entails reliably and efficiently recognizing the number of items in small collections and labeling them with the appropriate number word. The use of "one," "two," "three" in conjunction with seeing examples and non-examples of each can help 2- and 3-year-olds construct an increasingly reliable and accurate concept of the "intuitive numbers" *one*, *two*, and *three*—an understanding of oneness, twoness, and threeness. Consider, for example, constructing an understanding of "two":

- By seeing various examples of pairs, such as □□, □□, □ □ , and □□ , all labeled "two," young children can recognize that the appearance of the items in the collections is not important (shape and color are irrelevant to number). It can also provide them a label ("two") for their intuitive idea of *plurality* (more than one item).
- Seeing non-examples of pairs, such as □, □□□, □ , and □□□ labeled as "not two" or with another number word can help them define the boundaries of the concept of *two*.

The key instructional implications are that a basic understanding of cardinal number is not innate, nor does it unfold automatically (cf. Dehaene[15]).[14,16] Parents and preschool teachers are

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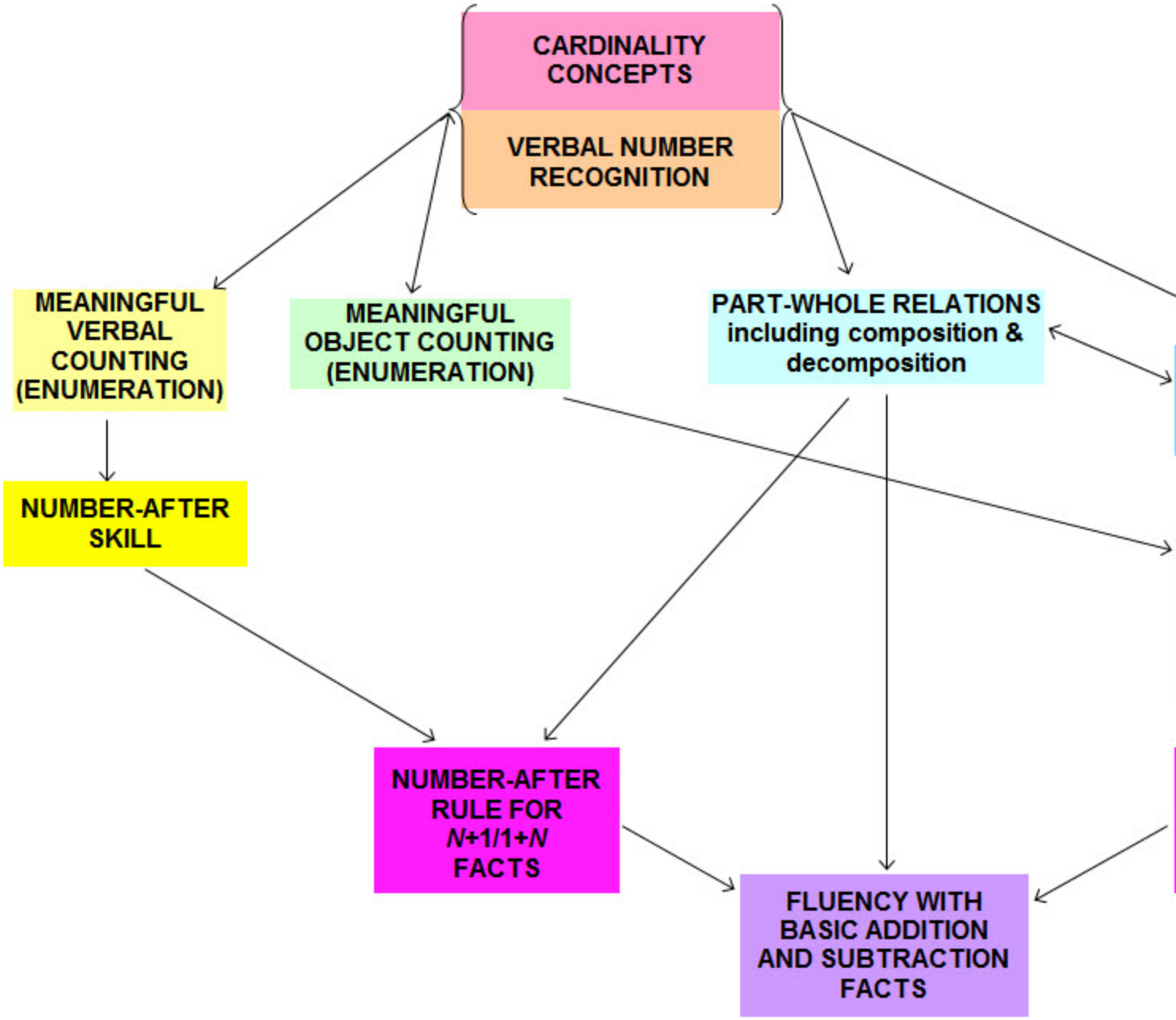
important in providing the experiences and feedback needed to construct number concepts. They should take advantage of meaningful everyday situations to label (and encourage children) to label small collections (e.g., “How many feet do you have?” “So, you need two shoes, not one.” “You may take one cookie, not two cookies.”). Some children enter kindergarten without being able to recognize all the intuitive numbers. Such children are seriously at risk for school failure and need intensive remedial work. Kindergarten screening should check for whether children can immediately recognize collections of one to three items and be able to distinguish them from somewhat larger collections of four or five.

As **Figure 1** illustrates, the co-evolution of cardinal concepts of the intuitive numbers and the skill of VNR can provide a basis for a wide variety of number, counting, and arithmetic concepts and skills—including fluency with basic addition and subtraction facts. Small-number concepts and VNR can provide a basis for meaningful verbal counting. Recognition of the intuitive numbers can help children literally see that a collection labeled “two” has more items than a collection labeled “one” and that a collection labeled “three” has more items than a collection labeled “two.” This basic ordinal understanding of number, in turn, can help children recognize that the order of number words matters when we count (the *stable order principle*) and that the number word sequence (“one, two, three…”) represents increasingly larger collections. As a child becomes familiar with the counting sequence, they develop the ability to start at any point in the counting sequence and (efficiently) state the next number word in the sequence (number-after skill) instead of counting from “one.”

The ability to automatically cite the number after another number in counting sequence, can be the basis for the insight that adding “one” to a number results in a larger number and, more specifically, the number-after rule for $n+1/1+n$ facts. When adding “one”, the sum is the number after the other number in the counting sequence (e.g., the sum of 7+1 is the number after “seven” when we count or “eight”).

Figure 1. Learning Trajectory of Some Key Number, Counting, & Arithmetic Concepts and Skills

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This reasoning strategy can enable children to efficiently deduce the sum of any such combination for which they know the counting sequence, even those not previously practiced including large multi-digit facts such as 28+1, 128+1, or 1,000,128+1. In time, this reasoning strategy becomes automatic—can be applied efficiently, without deliberation (i.e., becomes a component in the retrieval network). In other words, it becomes the basis for fact fluency with the n+1/1+n combinations.

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VNR, and the cardinal concept of number it embodies, can be a basis for meaningful object counting.[17] Children who can subitize collections up to "four" are more likely to benefit from adult efforts to model and teach object counting than those who cannot. When modeling involves counting and then labeling a collection within a child's subitizing range, they are also more likely to recognize the purpose of object counting (as another way of determining a collection's total) and the rationale for object-counting procedures (e.g., the reason why others emphasize or repeat the last number word used in the counting process is because it represents the collection's total).[18] Meaningful object counting is necessary for the invention of counting strategies (with objects or number words) to determine sums and differences. As these strategies become efficient, attention is freed to discover patterns and relations; these mathematical regularities, in turn, can serve as the basis for reasoning strategies (i.e., using known facts and relations to deduce the answer of an unknown combination). As these reasoning strategies become automatic, they can serve as one of the retrieval strategies for efficiently producing answers from a memory or retrieval network.

VNR can enable a child to see one & one as *two*, one & one & one as *three*, or two & one as *three* and the reverse (e.g., *three* as one & one & one or as two & one). The child thus constructs an understanding of composition and decomposition (a whole can be built up from, or broken down into, individual parts, often in different ways). Repeatedly seeing the composition and decomposition of *two* and *three* can lead to fact fluency with the simplest addition and subtraction facts (e.g., "one and one is two," "two and one is three," and "two take away one is one"). Repeatedly decomposing *four* and *five* with feedback (e.g., labeling a collection of four as "two and two", and hearing another person confirm, "Yes, two and two makes four") can lead to fact fluency with the simplest sums to five and is one way of discovering the number-after rule for n+1/1+n combinations (discussed earlier).

The concept of cardinality, VNR, and the concepts of composition and decomposition can together provide the basis for constructing a basic concept of addition and subtraction. For example, by adding an item to a collection of two items, a child can literally see that the original collection has been transformed into a larger collection of *three*. These competencies can also provide a basis for constructing a relatively concrete, and even a relatively abstract, understanding of the following arithmetic concepts[19]:

- Concept of subtractive negation. For example, when children recognize that if you have two blocks and take away two blocks this leaves none, they may induce the pattern that *any*

number take away itself leaves nothing.

- Concept of additive and subtractive identity. For example, when children recognize that two blocks take away none leaves two blocks, they may induce the regularity that if none is taken from any number, *the number will remain unchanged*. The concepts of subtractive negation and subtractive identity can provide a basis for fact fluency with the $n - n = 0$ and $n - 0 = n$ families of subtraction facts, respectively.

A weak number sense, then, can interfere with achieving fact fluency and other aspects of mathematical achievement. For example, Mazzocco and Thompson[20] found that preschoolers' performance on the following four items of the Test of Early Mathematics Ability—Second Edition (TEMA-2) was predictive of which children would have mathematical difficulties in second and third grade: meaningful object counting (recognizing that the last number word used in the counting process indicates the total), cardinality, comparing of one-digit numbers (e.g., Which is more five or four?), mentally adding one-digit numbers, and reading one-digit one numerals. Note that verbal number recognition of the intuitive numbers is a foundation for the first three skills and meaningful learning of the fourth.

Question 4. The basis for helping students build both number sense in general and fact fluency in particular is creating opportunities for them to discover patterns and relations. For example, a child who has learned the "doubles," such as 5+5=10 and 6+6=12, in a meaningful manner (e.g., the child recognizes that the sums of this family are all even or count-by-two numbers) can use this knowledge to reason out the sums of unknown doubles-plus-one facts, such as 5+6 or 7+6.

To be developmentally appropriate, such learning opportunities should be purposeful, meaningful, and inquiry based.[21]

- Instruction should be purposeful and engaging to children. This can be achieved by embedding instruction in structured play (e.g., playing a game that involves throwing a die can help children learn to recognize regular patterns of one to six). Music and art lessons can serve as natural vehicles for thinking about patterns, numbers, and shapes (e.g., keeping a beat of two or three; drawing groups of balloons). Parents and teachers can take advantage of numerous everyday situations (e.g., "How many feet do you have? …So, how many socks should you get from your sock drawer?"). Children's questions can be an important source of purposeful instruction.

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- Instruction should be meaningful to children, building gradually on (and being connected to) what they know. A meaningful goal for adults working with 2-year-olds is to have children recognize two. Pushing them too fast to recognize larger numbers such as *four* can be overwhelming, causing them to melt down (become inattentive or aggressive, guess wildly, or otherwise disengage from the activity).
- Instruction should be inquiry based, or thought provoking, to the extent possible. Instead of simply feeding children information, parents and teachers should give children an opportunity to think about a problem or task, make conjectures (educated guesses), devise their own strategies or deduce their own answer.

The various points above are illustrated by the case of Alice[22] and Lukas.[23]

- *The case of Alice.* The 2.5-year-old had for several months been able to recognize one, two, or three things. So, her parents wanted to expand her number range to four, which was now just outside her range of competence. Instead of simply labeling collections of four for her, they asked her about collections of four. Alice often responded by decomposing the unrecognizable collections into two familiar collections of two. Her parents then built on her response by saying, "Two and two is four." At 30 months of age, *shown a picture of four puppies, Alice put two fingers of her left hand on two dogs and said, "Two." While maintaining this posture, she placed two fingers of her right hand on the other two puppies and said, "Two." She then used the known relation "2 and 2 makes 4" (learned from her parents) to specify the cardinal value of the collection.*
- *The case of Lukas*. In the context of a computer-based math game, Lukas was presented 6+6. He determined the sum by counting. Shortly afterward, he was presented 7+7. He smiled and answered quickly, "Thirteen." When the computer feedback indicated the sum was 14, he seemed puzzled. A couple of items later, he was presented 8+8 and noted, "I was going to say 15, because 7+7 was 14. But before 6+6 was 12, I thought for sure that 7+7 would be 13 but it was 14. So, I'm going to say 8+8 is 16."

The Case of Fostering Fluency with Subtraction Facts: The Long View

To illustrate the implications of the recent research previously discussed, consider the case of promoting fluency with the more challenging facts basic subtraction facts such as 8–5 and 15–7. Mathematics educators, textbook publishers, and educational policy makers often routinely

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recommend helping children learn such facts by helping them learn a subtraction-as-addition reasoning strategy (e.g., for 8–5, think: “What added to five makes eight?”).[24,25] The short view, which is too commonly practiced, is to impose (e.g., demonstrate or illustrate) the reasoning strategy and perhaps attempt a brief explanation of it. Limited practice with the strategy is then used to promote its automaticity. A serious limitation of the short view is that many children do *not understand* the strategy. This can result in memorizing it correctly but not applying when appropriate or forgetting it altogether, memorizing it by rote incorrectly, or not making any effort to memorize it at all.

The long view is that fluency with the subtraction-as-addition strategy cannot be promoted in days, weeks, months, or even a year. Children’s informal and formal experiences often leads them to believe that addition and subtraction are unrelated operations and that knowledge of one cannot help with thinking about the other. The key to achieving fluency with basic difference *meaningfully* and *effectively* is (a) discover how the operations of addition and subtraction are related, (b) achieve fluency with related sums, and (c) then practice using this integrated knowledge until it becomes automatic.[26,27] As the learning trajectory depicted in Figure 2 indicates, the process of building the number sense for such a path to fluency is a gradual and begins in the preschool years.

- As previously noted, VNR provides a basis for an informal understanding of addition as a means for making a collection larger and subtraction as making a collection smaller (Concept 1 in Figure 2). These informal concepts of addition and subtraction provide a basis for understanding and (informally) solving word problems and symbolic expressions such as 7–4 and equations such as 7–4 = ?
- VNR can also provide a basis for experiences with empirical inversion—situations where a few items are added to (removed from) a small collection, the same number of items are then removed (or added), and the collection is restored to its original number). Such experiences can help children discover the *undoing concept*: addition and subtraction are related because adding and then subtracting the same number of items or vice versa leaves the original number of items (Concept 2 in Figure 4). This informal undoing concept provides a basis for understanding formal (written) representations of the concept, such as 7+4–4 = 7 and recognizing the *shared-numbers* concept (equations such as 7+4 = 11 and 11-4 = 7 are related and share the same three numbers; Concept 4 in Figure 2).

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- VNR can also help children construct *informal composition and decomposition concepts* (Concept 3 in Figure 2), which are the basis for a more formal part-whole view of addition and subtraction (Concept 4 in Figure 2). It permits children to imagine that two (small and nearby) collections ("parts") can be thought as a single, larger collection ("whole") or that a larger whole collection can be thought of two smaller groups (parts). For example, seeing a dice roll of •• and ••□ as "two" and another "two" and also seeing (or hearing an older player) label it as "four" can help a child see that two (smaller) collections of "two" can be re-imagined as parts of the (larger) collection or whole of "four" and the whole "four" can be re-imagined as the smaller parts of "two" and "two." Moreover, also seeing a dice roll of • and ••• as "one" and a "three" *and* as "four" can lead to the insight that different combinations of smaller collections or parts can make the same larger number or whole or that a whole can be separated into smaller groups or parts. Once children learn to read written numbers, children can relate their informal knowledge of composition and decomposition to written expressions such as 2+2 or 1+3 and written equations such as 2+2 = 4 or 1+3 = 4 and construct the following formal concepts:
    - Specifically, basic informal concepts of composition and decomposition and part-whole ideas can provide a basis for a formal interpretation of an addition equation such as 1+3 = 4 as the parts 1 and 3 compose the larger whole 4 (as opposed to informal view a collection of one is made larger by adding three more) and 4–3 = 1 as the larger whole 4 is composed of the known part 3 and the unknown part 1 (Concept 5 in Figure 2: formal part-whole knowledge of addition and subtraction). Viewing addition and subtraction in terms of such a part-whole meaning supports the conclusion that subtracting a part from a whole leaves a part that is smaller than the whole
    - Even before constructing a formal part-whole view of addition and subtraction (Concept 5 in Figure 2), elements of informal composition and decomposition can help children construct a formal *shared-numbers concept*: An understanding that the written expressions 1+6, 2+5, 3+4, 4+3, 5+2, and 6+1 all have the same sum (whole) 7 and, conversely, the number (whole) 7 can also be represented by the expressions 1+6, 2+5, 3+4, 4+3, 5+2, and 6+1. For both reasons, 1+6, 2+5, 3+4, 4+3, 5+2, and 6+1 form a "family of sums." Importantly, this can lead to recognizing families of sums are related to families of differences and that all family members consist of the three same three numbers (Concept 4 in Figure 2: the shared-numbers

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concept).

As the learning trajectory depicted in Figure 2 suggests, deepening an understanding the operation of subtraction and its relation to addition, which strengthen the foundation for achieving fluency with basic differences, can be realized in kindergarten and grade 1.

- The *shared-number concept* can be underscored by using—as is done in *Everyday Mathematics* program—"fact triangles" (see, e.g., Figure 3 or—for a detailed discussion—Baroody[26]).
- *Formal part-whole knowledge of addition and subtraction* can be fostered, in part, by explicitly labeling the elements of a fact triangle as a "whole" or a "part" (e.g., using an asterisk to note the "whole" or labeling 3+4 = 7 as "the part three and the part four make the total seven"). Fact rectangles can provide a relatively concrete representation of part-whole relations (see, e.g., Figure 4 or—for a detailed discussion—Baroody[26]). Solving part-whole word problems (see, e.g., Figure 5) can be helpful also.
- Promoting both the shared-number concept and formal part-whole knowledge in an integrated manner can foster the *shared parts and whole concept*: An addition-subtraction family of facts share the same whole and parts. The understanding provides a basis for recognizing the complement principle discussed in the next paragraph.

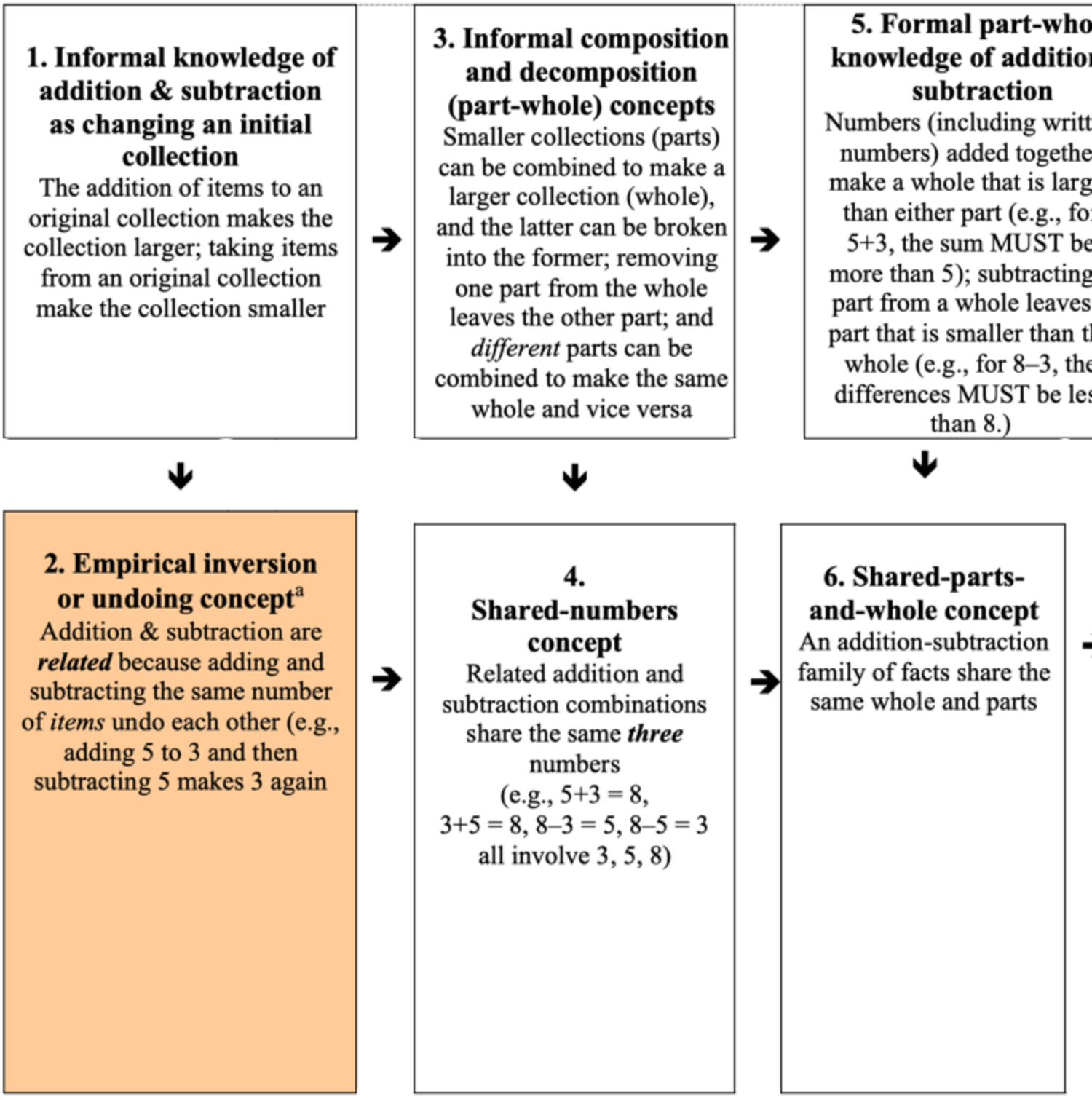
As the learning trajectory depicted in Figure 2 suggests, the final key elements for constructing subtraction-as-addition reasoning strategy and automatizing this strategy can now be achieved grade 2 or 3.

- *Fluency with basic sums to 18* can greatly facilitate using the subtraction-as-addition strategy to reason out consciously and then automatically subtraction facts.
- Discovering another key relation between addition and subtraction—*the complement principle* (e.g., if the parts 5 and 3 make the whole 8, then the whole 8 minus the part 3 leaves the part 5) —can provide a basis for understanding why the addition-as-subtraction strategy works and better enable children to internalize it.
- Practice using the subtraction-as-addition strategy can serve to automatize it and achieve *fluency with differences to 18*.

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Figure 2: Learning Trajectory for the Meaningful Development of the Subtraction-as-Addition Reasoning (Subtraction) Strategy

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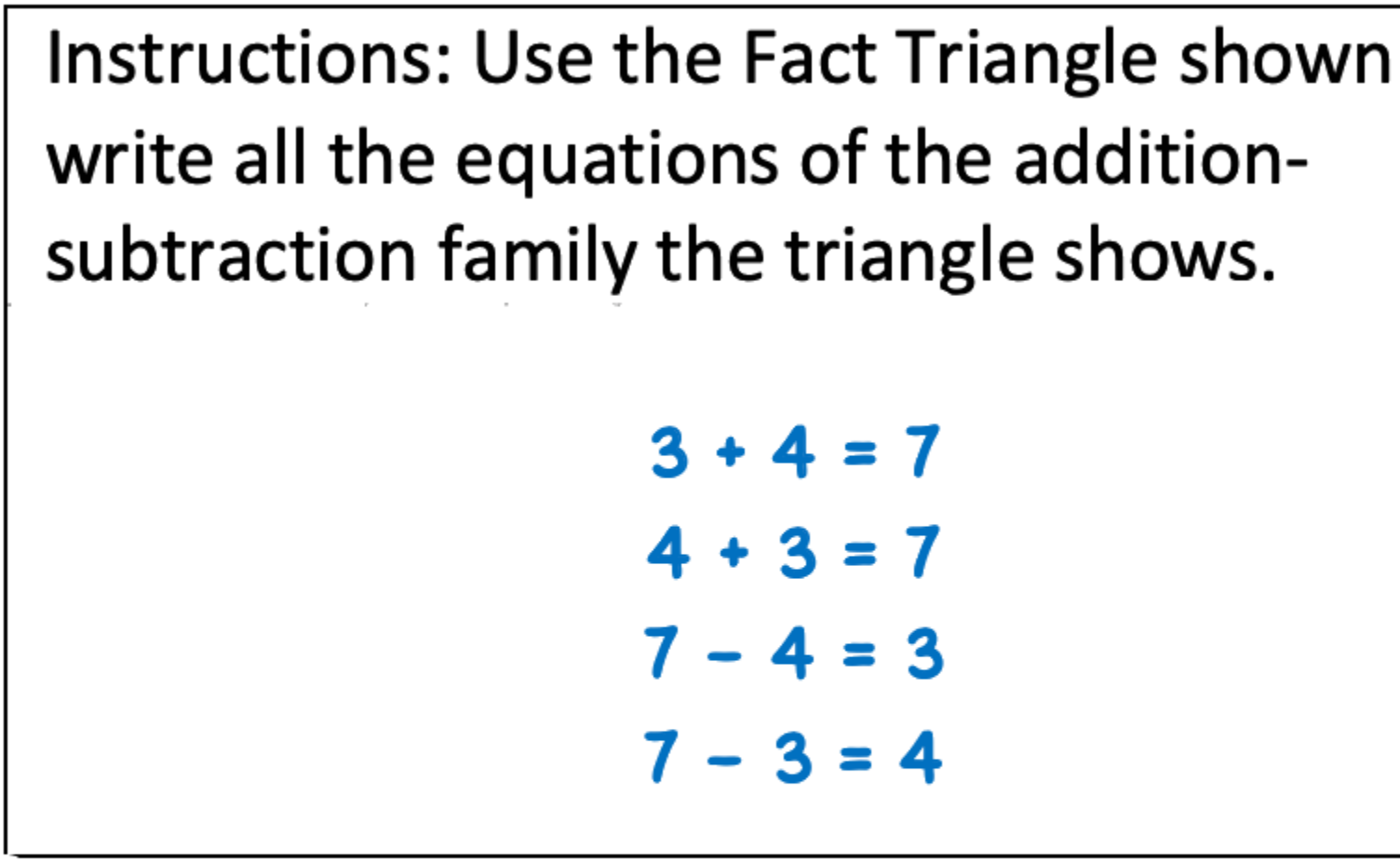


[1]All un-shaded cells are conceptual knowledge. The blue-shaded Cell 9 is conscious proced
integrated conceptual procedural knowledge of the complement principle and subtraction

[a]A general undoing concept can evolve into formal and explicit knowledge of the inverse pr
principle algebraically as $a + b - b = a$ or $a - b + b = a$.

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Figure 3. Fact Family Triangle



Note. * = the whole. When starting with t
sign is used (e.g., 3 → + → 4 → 7 creates
moving down, the minus (–) sign is used (

Figure 4. Fact Rectangles

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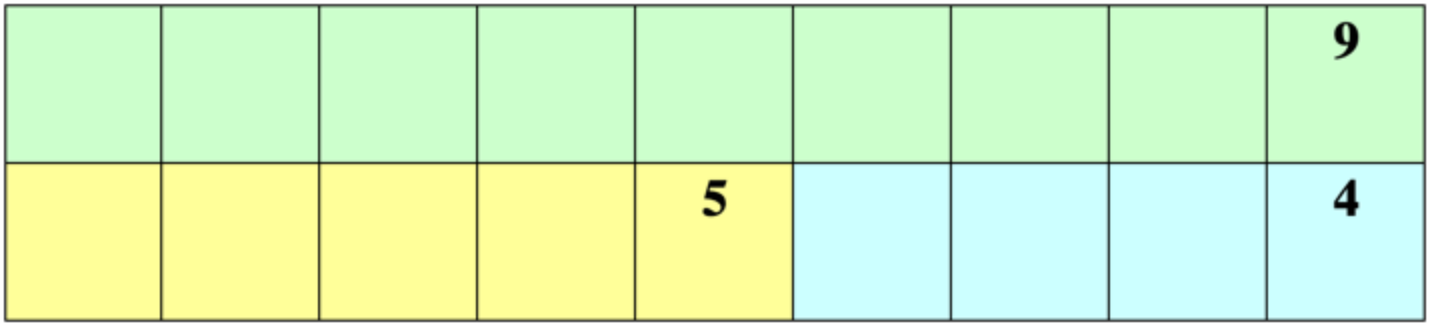
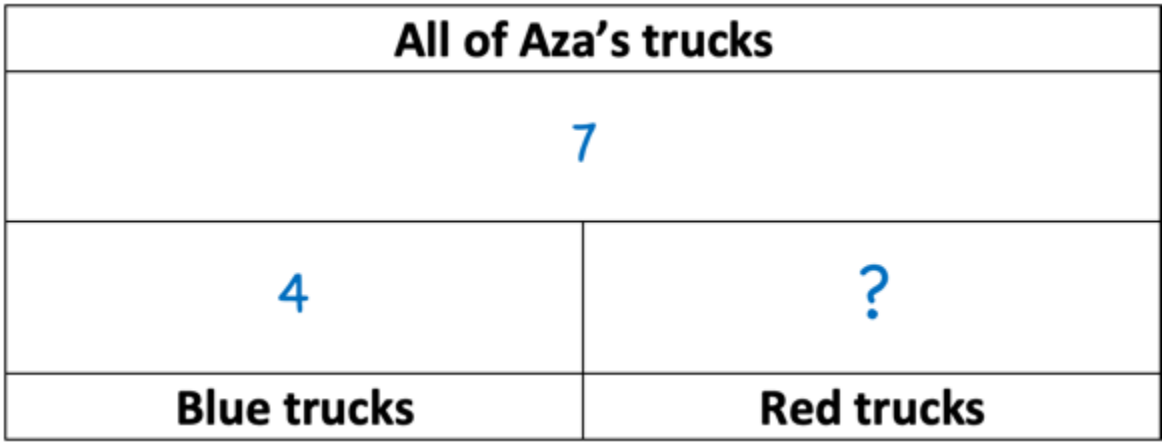


Figure 5. Example of Part-Part-Whole Word Problem

Word Problem

Aza had seven toy trucks. Four were blue and the rest were red. How many red trucks did Aza have?

Part-Part-Whole Picture



Equation: 4 + ? = 7 or 7 – 4 = ?

Answer: 3

Future Directions

Much still needs to be learned about preschoolers' mathematical development. Does VNR ability at two years of age predict readiness for kindergarten or mathematical achievement in school? If so, can intervention that focuses on examples and non-examples enable children at risk for academic failure to catch up with their peers? What other concepts or skills at two or three years

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of age might be predictive of readiness for kindergarten or mathematical achievement in school? How effective are the early childhood mathematics programs currently being developed?

Conclusions

Contrary to the beliefs of some early childhood educators, mathematics instruction for children as young as two years of age does make sense.[28,22,30,31] As **Figure 1** makes clear, this instruction should start with helping children construct a cardinal concept of the intuitive numbers and the skill of recognizing and labeling sets of one to three items with an appropriate number word. As **Figure 1 f**urther illustrates, these aspects of number knowledge are key to later numeracy and often lacking among children with mathematical disabilities.[32] For example, although memorizing the basic subtraction facts is often challenging, even difficult, or unobtainable for many children, it need not be *if* instruction helps children build number sense by discovering key arithmetic regularities at both the preschool and primary levels. Early instruction does not mean imposing knowledge on preschoolers, drilling them with flashcards, or otherwise having them memorize by rote arithmetic facts. Fostering number sense and fact fluency both should focus on helping children discover patterns and relations and encouraging their invention of reasoning strategies.

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Mathematics Instruction for Preschoolers

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Introduction

Teaching mathematics to young children, prior to formal school entry, is not a new practice. In fact, early childhood mathematics education (ECME) has been around in various forms for hundreds of years.[1] What has altered over time are opinions related to why ECME is important, what mathematics education should accomplish, and how (or whether) mathematics instruction should be provided for such young audiences.

Subject and Research Context

Is ECME necessary?

A concern among many early childhood experts, including educators and researchers, is the recent trend toward the "downward extension of schooling"[2] such that curricula, and the corresponding focus on assessment scores that were formally reserved for school-aged children, are now being pushed to preschool levels.[3] The motivation behind this downward push of curriculum appears to be largely political, with an increasing emphasis on early success, improving test scores, and closing gaps among specific minority and socio-economic groups.[4]

Despite the concern related to the downward extension of school-aged curricula in general, there are persuasive factors encouraging the presence of at least some type of mathematical instruction for preschoolers, or at least for some groups of preschoolers. As Ginsburg et al.point out, learning mathematics is "a 'natural' and developmentally appropriate activity for young children",[1] and through their everyday interactions with the world, many children develop informal concepts about space, quantity, size, patterns, and operations. Unfortunately, not all children have the same opportunities to build these informal, yet foundational, concepts of mathematics in their day-to-day lives. Subsequently, and because equity is such an important aspect of mathematics education, ECME seems particularly important for children from marginalized groups,[3] such as special needs children, English-as-additional-language (EAL)

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learners, and children from low socio-economic status (SES), unstable, or neglectful homes.[4]

Recent Research Results

Equity in education is one major argument for the presence of ECME, but intimately tied to equity is the aspect of helping young mathematical minds move from informal to formal concepts of mathematics, concepts that have names, principles and rules. Children's developing mathematical concepts, often building on informal experiences, can be represented as learning trajectories[5] that highlight how specific mathematical skills can build upon preceding experiences and inform subsequent steps. For example, learning the names, order and quantities of the "intuitive numbers" 1-3, and recognizing these values as sets of objects, number words, and as parts of wholes (e.g., three can be made up of 2 and 1 or 1 + 1 + 1), can help children develop an understanding of simple operations.[6] "Mathematizing," or providing appropriate mathematical experiences and enriching those experiences with mathematical vocabulary, can help connect children's early and naturally occurring curiosities and observations about math to later concepts in school.[3] Researchers have found evidence to suggest very early mathematical reasoning,[1,6,7] and ECME can help children formalize early concepts, make connections among related concepts, and provide the vocabulary and symbol systems necessary for mathematical communication and translation (for an example, see Baroody's paper[6]).

ECME may be important for reasons beyond equity and mathematization. In an analysis of six longitudinal studies, Duncan et al.[8] found that children's school-entry math skills predicated later academic performance more strongly than attentional, socioemotional or reading skills. Similarly, early difficulty with foundation mathematical concepts can have lasting effects as children progress through school. Given that math skills are so important for productive participation in the modern world (Platas L, unpublished data, 2006),[9] and that specific mathematical domains, such as algebra, can serve as a gatekeeper to higher education and career options,[10] early, equitable and appropriate mathematical experiences for all young children are of critical importance.

What is "appropriate" ECME?

Views differ with respect to what ECME should consist of and how it should be infused into preschoolers' lives, with a continuum that represents the amount of intervention or instruction proposed. On one end of the continuum is a very direct, didactic, and teacher-centered approach

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to ECME, while the other end of the spectrum represents a play-based, child-centered, non-didactic approach to ECME.[4] Individual children, and perhaps different groups of children, may benefit from varying levels of instruction throughout the continuum, and much research remains to be done to better understand best practices for all children and all aspects of mathematics. One example of a research-based mathematics curriculum for young children is Building Blocks, a program designed to support and enhance children's developing mathematical thinking (i.e., learning trajectories) through the use of computer games, everyday objects (i.e., manipulatives such as blocks), and print.[11] Building Blocks represents an attempt to align content and instructional activities with the learning trajectories of well-researched domains such as counting. The learning trajectories of other domains, such as measurement and patterning, are not yet well understood.[5]

Ginsburg et al.[1] described six components that should be present in all forms of ECME (e.g., programs such as Building Blocks), including environment, play, teachable moments, projects, curriculum, and intentional teaching. For example, regardless of where a particular mathematics curriculum falls on the playful–didactic continuum, environment is a vital component of early education. Specifically, providing preschool children with materials that inspire mathematical thinking, such as blocks, shapes, and puzzles, can facilitate the development of foundational skills such as patterning, making comparisons, and early numeracy. Another important component is that of the teachable moment: recognizing and capitalizing on children's spontaneous math-related discoveries by asking questions that require children to reflect and respond, by providing vocabulary and representational support, and by demonstrating extension activities that elaborate on and further support mathematical ideas.

Perhaps the most popular component of ECME in the current literature is play. Many proponents of play-based learning, or learning through play, argue that children learn a great deal when they discover mathematical ideas on their own in natural or minimally contrived situations.[12,13] Some argue that play is being taken out of preschools in reaction to the downward extension of schooling and testing,[14] and they provide data to suggest that children in early grades (including kindergarten) now spend far more time on test preparation than they do on play-based activities.[4] Even many educational toys appear marketed more toward early learning of academic concepts (i.e., literacy for toddlers) than toward playful learning per se. This approach may be driven in part by parents' views on the importance of early education for future academic success. Much research remains to be done on the impact of educational toys, technology, play (or lack thereof),

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and various ECME curricula on preschoolers' mathematical development.

Research Gaps and Implications

What are the barriers to effective early education?

Mathematics for preschool children is complicated by several factors, including political pressure (i.e., achievement scores, funding, varying curriculum standards), individual differences among preschoolers (i.e., individual children may benefit from different mathematical opportunities), ideological differences regarding education (i.e., playful–didactic continuum), and gaps in developmental research (i.e., uncertain learning trajectories for some mathematical concepts). Complicating ECME further are barriers that affect the implementation of mathematical instruction (regardless of curriculum), such as teachers' own fears or misunderstandings of mathematics. Unfortunately, many preschool educators lack training directly related to mathematics for young children (Platas L, unpublished data, 2006). Teachers need knowledge of what children know, knowledge of how children learn new concepts, knowledge of most effective teaching strategies, and the mathematical concepts themselves (Platas L, unpublished data, 2006).[3] Improving the mathematical training opportunities for early educators may help to improve the quality (and quantity) of mathematics instruction for young children.

Conclusion

The debate surrounding ECME does not appear to be about whether early exposure to mathematical experiences and ideas is important; the general consensus is that it is important. Rather, the issue is how, when, why and for whom specific approaches to ECME should be presented. Opinions differ regarding the amount of structure versus free-play and specific curriculum versus teachable moments. Yet as evidence accumulates regarding very young children's developing mathematical ideas (i.e., learning trajectories), attempts to align cognitive development with best instructional practices (or with the best environments to support natural mathematical discoveries) may help pave the way for equitable and appropriate mathematical experiences for all preschool children.

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