# Strategies to Enhance Young Children's Mathematical Development

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## Introduction and Subject

Evidence[1] suggests that 4 to 7% of the school-age population suffers from mathematics disability (MD). Although this prevalence rate is similar to the rate for reading disability, much less systematic study has been directed at MD.[2] Most available research describes the nature of the disorder; less work is available to inform the nature of effective prevention or remediation strategies. This relative neglect is problematic because MD is a serious public-health problem, leading to life-long difficulties in school and in the workplace and creating financial burdens on society. Mathematics competence, for example, accounts for variance in employment, income and work productivity even after intelligence and reading have been explained.[3]

## Research Context

In the primary grades (e.g. kindergarten through third grade), number combinations and word problems are two key dimensions of performance required to establish a strong foundation. Not surprisingly, therefore, these two aspects of math skills are persistent and severe and can cause difficulty for students with MD.[4] *Number combinations* are addition and subtraction problems with one-digit operands (e.g. 3+2=5). Competent performance involves automatic retrieval of answers from long-term memory. Individuals develop representations in long-term memory by pairing problems with answers using increasingly sophisticated counting and back-up strategies. *Word problems* are linguistically presented questions, sometimes including irrelevant information or charts/figures, for which answers require adding or subtracting of one- or two-digit numerals. Word problems also present persistent challenges for students with MD.

**Key Research Questions**

A key research question concerns what intervention strategies can be used to prevent difficulty or remediate deficits that develop in the primary grades.

**Recent Research Results**

To answer *number combination problems* (e.g. 2+3), typical children gradually develop procedural efficiency in counting. First, they count the two sets in their entirety (1, 2, 3, 4, 5); then they count from the first number (2, 3, 4, 5); and eventually they count from the larger number (3, 4, 5). As conceptual knowledge matures, children also develop backup strategies (2+3=[2+2]+1=4+1=5). As increasingly efficient counting and backup strategies help children consistently and quickly pair problems with correct answers, associations become established in long-term memory, and children gradually favour memory-based retrieval of answers.

Students with MD, however, manifest greater difficulty with counting5 and persist with immature backup strategies. So it is not surprising that they also fail to make the shift to memory-based retrieval of answers.6 When MD children do retrieve answers from memory, they commit more errors and manifest unsystematic retrieval speeds more than younger, academically normal counterparts.7 In fact, number combination deficits are a signature characteristic of students with MD. Prior work suggests the challenge of remediating this deficit with intermediate-grade students,8,9 which is unfortunate because number combination skill (NCS) appears to be foundational to higher-order performance.4 Given the foundational role NCS may play in the development of other math skills, along with the difficulties of remediation at higher grades, intervention may be important in the primary grades, when MD emerges.

Two competing approaches to intervention exist. With *conceptual instruction*, the teacher structures experiences to foster interconnected knowledge about quantities, with teacher explanations to guide students to correct understandings.10,11 The assumption is that NCS evolves from strong concepts, which lend meaning to the strings of numerals constituting arithmetic facts.12,13,14,15 The second intervention approach is *drill and practice*, whereby repeated pairings of problem stems with correct answers establish representations in long- term memory. Siegler's distribution of associations model16,17 accounts for the potential importance of both approaches. The model poses that early counting skills and backup strategies provide the basis for response accuracy. All results for a given problem constitute an individual's associations for that problem; so early errors interfere with the retrieval of number combinations later on. This suggests the need for better strategic thinking in the early stages (promoted by conceptual instruction) and the need for routine pairing of correct answers with problem stems (enhanced via drill and practice).

Unfortunately, there have been few investigations of intervention efficacy to develop NCS with children as early as in first, second or third grade. Most efficacy work is remedial, conducted with intermediate-age students, focuses exclusively on drill/practice and provides mixed results. 18,19,20 One of the few early intervention studies21 was a small pilot in first grade to assess the efficacy of computerized drill/practice. At-risk students ($n$ = 33) were randomly assigned to analogous drill and practice conditions in math or reading, stratifying by classroom (so that students in the same classrooms were in both conditions). The reading intervention served as the control for the math intervention. Students completed 50 to 54 sessions over 14 weeks and were pre- and post-tested. The math group improved significantly more than the reading group (ES = 0.92). In an ongoing remedial study with older students,22 drill and practice software was integrated with conceptual instruction. Using a multi-site randomized controlled field trial, 128 remedial students have so far completed intervention, and results reliably favour the experimental over the control group (ES = 0.73).

With respect to enhancing *word problem skill* (WPS), most research has assessed the value of planning and organization strategies with middle- and secondary-school students. For example, Montague and Bos23 assessed the effects of an eight-step metacognitive treatment with six adolescents with learning disabilities. Students were taught to read problems, paraphrase the problems aloud, graphically display known and unknown information, state the known and unknown information, hypothesize solution methods, estimate answers, calculate answers and check answers. Using a single-subject design, the researchers showed that this metacognitive treatment promoted WPS. With group design, Charles and Lester24 provided support for a similar approach among typically developing fifth and seventh graders.

The major contrasting intervention approach for developing WPS is schema-based instruction. According to Cooper and Sweller,25 students develop WPS by first mastering rules for solving problem types and then developing schemas to group problems into types that require similar solution strategies. The broader the schema, the greater the probability individuals will recognize connections between problems they have worked during instruction and novel problems. In experimental work at the intermediate grades, Jitendra et al.26 invoked schema-based instruction to enhance WPS with good success. We have extended that work to third grade, where the goal was to promote complex WPS. For each of four problem types, students were taught problem-solution rules. Then, with schema-based instruction, children were familiarized with the notion of transfer and taught to build schemas by showing them how superficial problem features change without altering problem-solution rules. In a series of randomized controlled trials, Fuchs et al. 27,28,29 provided empirical support for this approach, with large effect sizes (0.89- 2.14). More recently, Fuchs et al.30 extended this research program at third grade to address one-step change, equalize and compare word problems. Students with math and reading disability ($n$=40) were randomly assigned to schema-based instruction and control groups; results showed the efficacy of this approach with effect sizes of 0.77 to 1.25.

**Conclusions**

A theoretically supported approach, for which promising empirical evidence exists for promoting NCS, is conceptually-oriented instruction into which drill and practice on number combinations is integrated. For promoting WPS, the two major competing approaches are metacognitive instruction, with which teachers help students apply planning and organization strategies, and schema-based instruction. To date, however, few investigations of intervention efficacy have contrasted the two prominent approaches for promoting NCS or WPS, and inadequate work has been conducted at the primary grades. In addition, no studies of long-term maintenance have been conducted.

**Implications**

MD is a serious public-health problem, leading to life-long difficulties in school and in the workplace and creating financial burdens on society. In light of the serious negative outcomes associated with poor math performance, additional research is warranted to examine methods for prevention and remediation, especially in the primary grades. At present, research tentatively supports the use of conceptually-oriented instruction into which drill and practice on number combinations is integrated for addressing number combination difficulties. Metacognitive instruction and schema-based instruction represent promising strategies for promoting word problem skills.

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