## NUMERACY

# Early Numeracy: The Transition from Infancy to Early Childhood 

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## Introduction

By preschool age, most children exhibit a range of numeracy skills, including verbal skills, such as counting, and nonverbal skills, such as recognizing equivalence of object sets. ${ }^{1,2,3,4,5}$ Although researchers agree that these abilities are present in early childhood, they continue to debate when, and by what mechanisms, these abilities emerge. In other words, what are the developmental origins of verbal and nonverbal numerical competencies?

## Subject

Research on numeracy traditionally focused on verbal counting. However, the notion that numeracy might emerge in infancy and toddlerhood shifted the focus toward concepts that can be measured nonverbally. This shift expanded the range of behaviours included in early numeracy-a change that has direct implications for early childhood education and assessment. This shift also raised questions about the developmental origins of mathematics learning difficulties and gaps in mathematical achievement.

## Problems

Most children acquire basic symbolic number skills by 5 years of age, such as reciting the count list to 20 or more, ${ }^{1,2,4}$ using the count list to enumerate various sets, , $1,2,4,5$ understanding that the last word in a count stands for the numerosity of the set (i.e., Cardinal Word Principle or CWP), ${ }^{1,2,5,6}$ identifying written numerals, ${ }^{5}$ and judging ordinality of single-digit numerals. ${ }^{7}$ There is also emerging evidence that children can interpret multidigit numbers starting at 3 years of age-correctly judging, for example, which of two multidigit numerals is larger. ${ }^{8,9}$

Prior to mastery of symbolic skills, preschool children also exhibit understanding of quantitative relations on nonverbal measures, such as matching equivalent sets of objects, ${ }^{10}$ performing simple calculations with objects, ${ }^{11}$ or indicating which of two dot clouds has more. ${ }^{12}$ Children perform object-based number tasks earlier than they demonstrate similar understandings in verbal tasks. For example, preschoolers solve simple object-based addition and subtraction problems (e.g., $2+$ 2) years before they can solve analogous verbal problems. ${ }^{11,13}$ Similarly, children judge ordinality and equivalence in forced choice tasks earlier than they can compare the same sets verbally, via counting, with nonverbal competence emerging between $2-1 / 2$ and 3 years of age. ${ }^{11,14,15}$

A major research focus has been understanding the developmental origins of these nonverbal number concepts. Researchers have shown, using habituation and preferential looking methods, that infants are sensitive to quantity as well, ${ }^{16,17}$ with some studies demonstrating this sensitivity in newborns. ${ }^{18,19}$ Various proposals have linked individual variation in this early sensitivity to later numeracy and mathematics outcomes. However, open questions remain about the representations underlying this early sensitivity, how the representations themselves develop, and what role these representations may play in subsequent development.

## Research Context

One candidate for early nonverbal number representation is the Approximate Number System, or ANS—a representation proposed to underlie the discrimination of different set sizes-particularly large set sizes (e.g., 16 vs. 32). ${ }^{20,21}$ Although the ANS is thought to operate over discrete number, it is also inexact and ratio-dependent, similar to non-numerical dimensions such as surface area, meaning that quantities are easier to tell apart when their ratio is higher (e.g., 16 is easier to discriminate from 32 than from 24). ${ }^{22}$ The ANS is considered innately available because even newborns respond to variations in set sizes as long as the ratios are large enough. ${ }^{19}$ However,
research also shows that with age and schooling, the ANS becomes more precise. ${ }^{23,24,25,26}$

Another proposed nonverbal number representation is based on object individuation, also described as object tracking, mental models, or subitizing-the immediate perception of number in small quantities (e.g., 1 to 4 objects). ${ }^{11,21,27,28}$ On these accounts, children incidentally represent number when they differentiate objects in a scene and keep track of the objects' movements and spatial positions. Set size limits on object individuation have been explained by constraints on working memory ${ }^{29}$ or attention. ${ }^{27}$ Some have argued that like the ANS, object-based representations are an innate endowment, with ongoing debate about whether the two systems are distinct ${ }^{11,30}$ or simply different instantiations of the same evolutionarily primitive representational system. ${ }^{31}$ Still others have suggested object-based representations could emerge from experiences observing and manipulating objects without necessarily arising from an innate quantification system. ${ }^{32}$

A third contributor to early numeracy is exposure to number words and the verbal counting system. Prior to the advent of research on infant quantification, seminal research by Piaget suggested that children lacked a conceptual understanding of quantitative relations until well after they had mastered conventional counting ${ }^{33}$ and studies showed that children did not understand numeracy principles until after they had mastered counting procedures. ${ }^{34,35}$ Although subsequent research has shown that precounting children understand much more about quantities than Piaget claimed, symbolic number understanding remains a strong predictor of later mathematics achievement, ${ }^{36,37,38,39,40}$ and indeed, stronger than nonverbal quantification skills. 41,42,43,44 Research has also suggested children can extract information about numbers and their meanings from numeric symbols themselves, showing for example, that preschool children can match written multidigit numerals to multidigit number words and compare magnitudes of written multidigit numerals independent of performance using nonverbal measures. ${ }^{9}$

## Key Research Questions

Most researchers agree that children respond to changes in number early in life via nonverbal processes. Furthermore, there is general agreement about the stages of verbal number acquisition. Current research is now focused on the underlying nature of nonverbal quantification and whether variation in nonverbal processes is related to later mathematics achievement. In this research, investigators also consider whether children bootstrap between verbal and nonverbal quantification as they learn. ${ }^{45}$ Finally, there is growing interest in the verbal numeracy
environment at home and in preschool, and its connection to later child outcomes.

## Recent Research Results

## Early number discrimination and non-numerical quantitative dimensions

There is ongoing debate about whether infants' responses to quantitative changes are based on an awareness of discrete number per se, or one of many perceptual variables that correlate with discrete number, such as surface area, convex hull, brightness, duration, temporal density, and spatial frequency. ${ }^{46,47,48}$ Researchers have attempted to control these perceptual variables to obtain a clean test of numerical sensitivity, ${ }^{24,49,50}$ but it is difficult to control all of these perceptual variables simultaneously, as others have pointed out, ${ }^{46,47}$ leading some to suggest that future research should focus on ways to account for non-numerical responses rather than attempting to control them. ${ }^{46,50,51}$ Thus, it remains unclear whether infants' quantitative sensitivity is based on discrete number, as some have claimed, or a combination of other perceptual information that is correlated with discrete number. Similar issues arise in research testing whether infants respond to changes in quantity across dimensions-for example, learning to associate certain visual patterns with larger and smaller numerical sets and transferring this association to objects differing in size, ${ }^{52}$ or expecting that if quantitative pairs (e.g., number and spatial extent) both increase or decrease, they will both change in the same direction ${ }^{18}$-research which has led to the proposal that quantification arises from a generalized magnitude representation. Such a representation is one way to characterize an undifferentiated sense of quantity based on multiple input streams, but the claim that children can switch from one quantitative cue to another would require controls that can isolate each cue effectively.

## Making connections

Research has documented how children acquire several distinct verbal enumeration skills (e.g., counting, cardinality, ordinality), as well as how they represent quantities nonverbally. However, to achieve a coherent number concept, children must eventually make connections among these skills and representations (e.g., verbal number words, physical quantities, mental models). ${ }^{43,53,54,55,56,57}$ Small number words may play a critical role in children's first mappings because the quantities one, two, and three can be immediately perceived and represented nonverbally with less error than representations of larger quantities. Thus, small sets may offer clear perceptual referents that can be labeled with a number word. ${ }^{28,58,59,60}$

Once the labels for small sets have been learned, children are positioned to notice that the same words are used as labels and in counting, thereby discovering the Cardinal Word Principle (CWP)—the idea that the last word in a count stands for its cardinal number. In the absence of targeted instruction, most children naturally attain the CWP by age 4 years, but studies have shown the CWP can be induced from practice labeling small sets as well as instruction that juxtaposes counting and labeling. ${ }^{28,61}$ In the $n$-knower framework, CWP has been measured using the Give-n task (e.g., "Give me 5 counters."), and early research findings suggested children learn number-to-quantity mappings one by one and in order, prior to making the connection between counting and cardinality, which itself is followed by a rapid logical generalization of the CWP to all the numbers within a child's counting range. ${ }^{45,62,5}$ However, diary studies have reported correct use of small number words in certain contexts even earlier, as well as evidence that children may acquire these number meanings in a different order. ${ }^{63,64}$ Moreover, recent studies have raised questions about the validity of Give-n performance and the meaning of $n$-knower classifications based on it. ${ }^{65,66,67,68}$ Thus, although much has been learned about these important connections, key questions remain unresolved.

## Early predictors of mathematical achievement

Evidence of quantitative sensitivity in infancy has inspired researchers to examine how this sensitivity relates to acquisition of verbal numeracy in early childhood, as well as eventual mathematical achievement in school. Some have argued that the ANS provides a representational foundation for acquisition of later symbolic numeracy and mathematics skills ${ }^{21,45}$ and longitudinal studies linking ANS acuity in infancy and preschool to later mathematical achievement in childhood and adolescence seem to bolster this claim. ${ }^{69,70,71,72}$ However, other studies examining longitudinal and concurrent associations have failed to find evidence connecting ANS acuity to mathematics achievement, ${ }^{73,74,75,76,77}$ and indeed, accumulating neurological and behavioural evidence points to separate mechanisms. ${ }^{12,26,78}$ Finally, when children acquire symbolic numeracy skills, ANS acuity improves concurrently, perhaps as a result. ${ }^{25,79}$ Thus, if ANS and symbolic mathematics skills are causally related, the relation could be from symbolic number to ANS rather than the reverse, or perhaps, bidirectional.

Similar patterns have been reported for spontaneous focusing on number (SFON)-the tendency of children to notice exact number in their daily experiences. ${ }^{80}$ Tests of SFON carefully avoid verbal number cues in order to tap children's self-directed attention to numerosity, but because children in these studies are generally preschool aged or older, ${ }^{80,81}$ it is unclear whether the
mechanism driving SFON is nonverbal quantification (e.g., object individuation), verbal counting, or both. Concurrent correlational studies indicate strong associations between SFON tendencies and verbal numeracy, ${ }^{80,82}$ and longitudinal studies demonstrate that performance on SFON measures in early childhood is correlated with symbolic number knowledge in elementary school; ${ }^{83,84}$ however, whereas both SFON and symbolic numeracy predict subsequent mathematics achievement, performance on symbolic number tasks is the stronger predictor. ${ }^{85}$ Also, attempts to improve SFON have been successful when interventions included symbolic number activities, ${ }^{86,87}$ suggesting that SFON itself may be driven by symbolic numeracy acquisition, rather than the reverse. Additional research using interventions based on nonverbal activities is needed to draw firm conclusions, but early symbolic numeracy remains the clearest and most potent predictor of later mathematics achievement.

## Home Numeracy Environment

Acquisition of children's first numeracy skills takes place largely in the family home, so the number-related activities of children and their caregivers have received increasing attention. ${ }^{88,89}$ Most research on this topic has used either parent report of numeracy activities ${ }^{90,91,92,93,94}$ or coding parent speech from direct observations. ${ }^{95,96,97}$ Studies have demonstrated an association between the frequency of home numeracy activities based on parent report and children's numeracy outcomes, ${ }^{90,91}$ though this association is not always obtained. ${ }^{89,92,93}$ Existing studies also indicate that although parents talk about number infrequently, ${ }^{95}$ even when activities are designed to elicit such talk, ${ }^{60,97}$ there are significant associations between the frequency of incidental number talk and children's numeracy outcomes. ${ }^{90,95,96,97,98}$ Child outcomes have also been linked to variation in qualitative differences, such as conversation length, ${ }^{99}$ and focusing on large set sizes (e.g., 4-10) or advanced concepts such as cardinality. ${ }^{97,100,101} \mathrm{~A}$ few observational studies have targeted infancy in particular, demonstrating that parental number talk is present at the youngest ages observed to date (i.e., 12 to 14 months). ${ }^{95,96}$ Thus, although infants are themselves nonverbal, their emerging understandings of numeracy may be shaped by exposure to verbal numeracy early on. ${ }^{32}$

## Research Gaps

Although research has generated extensive information about developmental changes in various quantitative skills, such as the CWP, SFON, and nonverbal set size discriminations, less is known about the mechanisms that drive these changes, and particularly, the mechanisms by which children make connections among various concepts and representations to achieve a coherent
sense of number. Related to this issue, more research is needed to test proposed mechanisms experimentally, by providing inputs that are consistent with hypotheses about developmental mechanism. For example, though it has been argued that small set sizes offer an opportunity to unite verbal and nonverbal quantification, the next step is to demonstrate that this is the case experimentally. Intervention studies that test the effects of specific input types may also be helpful in this regard. Similarly, more research examining the relations between verbal number and nonverbal number are needed to determine what directions of influence are at play, at what ages, and under what conditions.

Another persistent issue that remains unresolved is whether nonverbal quantification is based on discrete number or attention to non-numerical variables, such as surface area. Although researchers have focused on attempts to control these non-numerical variables, a promising alternative may be to design measures that account for non-numerical responses rather than attempting to control for them. ${ }^{46,50,51}$

Finally, intriguing new research about the home numeracy environment as well as the origins of multidigit number concepts have raised a host of new questions that bear investigation. For example, most studies of children's home numeracy environment have focused on preschool age, but much could be gained by tracing these experiences back into infancy, particularly given the longstanding evidence of nonverbal quantification in this age range. Are infants directed to attend to number much earlier in life than we have documented to date? If so, how might this change our understanding of SFON, for example. Similarly, the unexpectedly early acquisition of multidigit number meanings raises new questions about the presence of multidigit numeracy in parents' number talk, as well as whether variation in these informal insights is related to subsequent mathematics outcomes. Interventions targeting either the home numeracy environment, early multidigit numeracy, or both, would be exciting new directions for future research.

## Conclusions

Evidence of numerical competence in infants has raised intriguing questions about the origins of numeracy and the conceptual resources young children use to acquire verbal counting. However, further research is needed to reveal precisely how this infant competence connects to subsequent nonverbal and verbal development and whether these mechanisms can be leveraged to help all children enter school with a strong foundation of numeracy concepts.

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