

NUMERACY

Early Predictors of Mathematics Achievement and Mathematics Learning Difficulties

Nancy C. Jordan, PhD

University of Delaware, USA

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Introduction

Mathematics difficulties are widespread. Up to 10% of students are diagnosed with a learning disability in mathematics at some point in their school careers.^{1,2} Many more learners struggle in mathematics without a formal diagnosis. Mathematics difficulties are persistent, and students who have difficulties may never catch up to their normally achieving peers.

Subject

Foundations for mathematics achievement are established before children enter primary school.^{3,4} Identification of key predictors of mathematics outcomes provides support for screening, intervention and progress monitoring before children fall seriously behind in school.

Problem

The consequences of poor mathematics achievement are serious for everyday functioning, educational attainment, and career advancement.⁵ Mathematics competence is necessary for entry into STEM (science, technology, engineering and mathematics) disciplines in college and for STEM occupations.⁶ There are large group differences in mathematics achievement related to socioeconomic status⁷ as well as individual differences in fundamental learning abilities.⁸ These differences are already present in early childhood and increase over the course of schooling.

Research Context

Longitudinal studies of characteristics of children with mathematics difficulties have identified important targets for intervention. Most children enter school with *number sense* that is relevant to learning school mathematics. Preverbal components of number (e.g., exact representations of small quantities and approximate representations of larger quantities) develop in infancy.^{9,10,11} Although these primary foundations are thought to underlie learning of conventional mathematics skills, they are not sufficient. Most children with difficulties in mathematics are characterized by weaknesses in secondary symbolic number sense related to whole numbers, number relations and number operations¹² – areas that are malleable and influenced by experience.¹³

Key Research Questions

In the area of literacy, reliable and valid early screening measures have led to effective interventions and supports in early childhood and later.¹⁴ Intermediate measures closely tied to reading (e.g., knowledge of letter sounds) are more predictive of reading achievement than are more general competencies. Similarly, in the numeracy area, early competencies that are allied with the mathematics children are required to do in school are most predictive of mathematics achievement and difficulties.¹⁵ Key longitudinal predictors of mathematics performance need to be identified for early screening.

Recent Research Results

Early number competencies are important for setting children's achievement trajectories in mathematics.^{16,17} Mathematics difficulties and disabilities have their roots in weak number sense.^{18,19} Children with developmental dyscalculia, a severe form of mathematics disability, are characterized by deficits in recognizing and comparing numbers and in counting and enumerating sets of objects.¹⁸

Longitudinal predictors

Short-term longitudinal studies (from the beginning to the end of the kindergarten year) reveal that numeracy indicators of counting, quantity discrimination, and number naming are moderate to strong predictors of mathematics achievement.^{20,21,22} Moreover, performance on numeracy indicators in preschool predicts performance on similar measures in kindergarten.²³ Low-income children enter kindergarten well behind their middle-income peers on numeracy indicators, and this gap does not narrow during the course of the school year.¹²

Longitudinal studies over multiple time points, from the beginning of kindergarten through the end of Grade 3, suggest that foundational number sense supports the learning of complex mathematics associated with computation as well as applied problem solving.^{15,17,24,25} Kindergarten numeracy related to counting, numerical magnitude comparisons, nonverbal calculation, and verbal arithmetic predict mathematics level and rate of achievement in Grades 1 through 3. The low mathematics achievement of high-risk, low-income students is mediated by early number competence. Number competence also predicts later mathematics outcomes over and above IQ variables.²⁶ Kindergarten competence with simple arithmetic calculations involving addition and subtraction is most predictive of later mathematics achievement. Because early number competencies are achievable in most children⁴ their intermediate effects provide direction for early intervention.

Underlying pathways

Three underlying cognitive pathways—quantitative, linguistic and spatial—contribute independently to number competencies in preschool and kindergarten.²⁷ Linguistic skills are unique predictors of number naming, whereas quantitative skills are unique predictors of nonverbal calculation; spatial attention is a distinct predictor of both types of early numeracy. These precursor pathways relate differently to mathematical outcomes two years later (e.g., the linguistic but not the quantitative pathway is uniquely predictive of geometry and measurement concepts). A pathway model may explain why learners perform relatively well in one area of mathematics but not in another.²⁸

Research Gaps

Screening tools for identifying foundational number competencies in preschool and kindergarten need to be developed and validated for use in schools, clinics and other educational settings. Interventions for children with, or who are at risk for, mathematics learning difficulties should be

devised and evaluated through randomized controlled studies. In particular, researchers must study how gains in specific areas of number competence can be achieved most effectively and whether gains can be sustained over time and generalized to mathematics learning. Further it is important to differentiate more and less effective methods of increasing number competence.

Conclusions

Difficulties with mathematics are pervasive and can have lifelong consequences. Foundational number competencies develop before Grade 1 and are highly predictive of mathematics achievement and difficulties. Higher levels of kindergarten number competence predict statistically significant and substantively meaningful performance in mathematics applications and computation at the end of Grade 3. Symbolic number competencies associated with whole number relations, and operations are particularly important. Number competence depends on language abilities (e.g., knowing number names), as well as on quantitative and spatial knowledge (combining and separating sets). Although there are poorer long-term outcomes for low-income children than for middle-income children, mathematics achievement is moderated by early number competencies. Low-income children enter school with relatively few number-related experiences,²⁹ which contributes to their disadvantage. The intermediate effect of number competence on mathematics achievement suggests that it should be emphasized in preschool and kindergarten. Overall, early number sense is critical for setting mathematics trajectories in mathematics throughout elementary school.

Implications for Parents, Service, and Policy

In today's schools, mathematics learning difficulties and disabilities often are not identified before Grade 4. Early interventions in mathematics are far less common than are those for reading. Kindergarten teachers should screen students for numeracy difficulties, similar to the way that most screen for early literacy difficulties. Preschools and kindergartens should provide mathematics experiences and instruction in number, number relations and number operations.⁴ This *number core* should emphasize the number word list, counting principles related to cardinality and one-to-one correspondence, comparing set sizes, and joining and separating sets. Number lists and simple board games using number lists can help children make sense of quantities.³⁰ Curriculum developers in early childhood should focus their materials on these core number foundations. Children in schools serving low-income communities are especially at risk for learning difficulties with mathematics. Low-income children enter kindergarten well behind

their middle-income counterparts. Early interventions can help all children build the foundations they need to achieve in mathematics.

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