ENCYCLOPEDIA
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Development

Early Predictors of Mathematics Achievement and Mathematics Learning Difficulties

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Introduction

Mathematics difficulties are widespread. Up to 10% of students are diagnosed with a learning disability in mathematics at some point in their school careers.[1,2] Many more learners struggle in mathematics without a formal diagnosis. Mathematics difficulties are persistent, and students who have difficulties may never catch up to their normally achieving peers without intervention.

Subject

Foundations for mathematics achievement are established before children enter primary school.[3,4] Identification of key predictors of mathematics outcomes provides support for screening, intervention and progress monitoring before children fall seriously behind in school.

Problem

The consequences of poor mathematics achievement are serious for everyday functioning, educational attainment, and career advancement.[5] Mathematics competence is necessary for entry into STEM (science, technology, engineering and mathematics) disciplines in college and for STEM-related occupations.[6] There are large group differences in mathematics achievement related to socioeconomic status[7] as well as individual differences in general learning abilities.[8] These disparities are already present in early childhood and increase over the course of schooling.

Research Context

Longitudinal studies of characteristics of children with mathematics difficulties have identified important targets for intervention. Most children enter school with *number sense* that is relevant to learning school mathematics. Preverbal components of number (e.g., perceiving exact representations of small sets of objects and approximate representations of larger sets) develop in infancy.[9,10,11,12] Although these primary foundations are thought to underlie learning of conventional mathematics skills, they are not sufficient. Most children with difficulties in mathematics show weaknesses in number sense related to knowledge of number, number relations, and number operations[4,13] – strands of number sense are malleable and influenced by experience.[14] Early number relates to knowledge of oral and written number and counting concepts, such as one-to-one correspondence and cardinality. Number relations involve understanding of numerical magnitudes on the number line. Number operations relates to transforming quantities through addition and subtraction.[4,15]

Key Research Questions

Early competencies that are aligned with the mathematics children are required to do in school are most predictive of mathematics achievement and difficulties.[16] Assessment tools and interventions need to be refined to help children develop key number sense concepts. Identifying pathways and factors related to number sense development is needed to guide the creation of early childhood interventions for those at-risk for developing mathematics learning difficulties in school.

Recent Research Results

Early number sense sets children’s achievement trajectories in mathematics.[16,17,18,19,20] Mathematics difficulties and disabilities have their roots in poorly developed number sense.[21,22,23] Children with developmental dyscalculia, a severe form of mathematics disability, are characterized by deficits

in counting and enumerating sets of objects and in recognizing and comparing numbers.[21] Such deficits lead to poor arithmetic fluency, a hallmark skill in the primary grades.

Number sense as a predictor of later mathematics achievement and difficulty

Studies provide empirical evidence of a multifactor early number sense model consisting of specific strands of number, number relations, and number operations understanding.[24,25] Early screening tools developed using this model accurately identify children at risk for mathematics learning difficulty and disability.[26,27,28] Predictive relations may differ by level of number sense in preschool. Number strand skills predict later mathematics achievement for children with low and intermediate achievement, but not high achievement; number relations strand skills predict later achievement for children of all math achievement levels; number operations strand skills predict later math achievement for children with intermediate and high achievement, but not low achievement.[29]

Low-income children enter kindergarten well behind their middle-income peers on most symbolic numeracy indicators, and this gap does not narrow during the school year.[13] Longitudinal studies over multiple time points, from the beginning of kindergarten through the end of Grade 3, suggest that foundational number sense supports the learning of complex mathematics associated with computation as well as applied problem solving and fluency.[16,20,30,31] The low mathematics achievement of high-risk, low-income students is mediated by kindergarten number sense. Because early number competencies are achievable in most children[4] their intermediate effects provide clear directions for early intervention

Type of quantity representation and set size

The size of a quantity or set along with the way it is presented to a child (e.g., non-symbolic or symbolic) affects children's reasoning about numbers. Children's ability to map written numerals to the quantities they represent is critical for learning more complex number sense skills.[32] Non-symbolic quantity representations (e.g., which of 2 sets of dots has more, without counting) scaffold the development of symbolic understanding (which of 2 numerals is bigger), but only for small sets (i.e., 4 or less).[33] The results suggest that children may be able to engage in both symbolic and non-symbolic number sense activities across strands (number, number relations, and number operations) with small set sizes. An intervention in which children engaged in a variety of number sense skills with small set sizes before cycling to a similar sequence with larger

set sizes was successful in kindergartners at-risk for later mathematics learning difficulties.[34]

Developmental pathways

Research has revealed patterns of individual differences in early number sense development. There are empirically distinct developmental pathways in number, number relations, and number operations for preschoolers across the school year,[35,36] which predict mathematics achievement in Grades 1 and 3.[35] Low receptive vocabulary[35,36] and visual-spatial working memory skills[34] predict membership in a consistently low developmental pathway, emphasizing the importance of domain-general learning skills for early numeracy development.[37] Contextual factors such as children's home learning environment also relate to individual differences in early numerical development.[38]

Research Gaps

Additional work is needed to consider how the number sense strands of number, number relations, and number operations work together during the early childhood period. Researchers must also consider how set size and level of representation constrain the development of number sense, including for children at-risk for mathematics learning difficulties. Interventions that target and weave together the strands of number sense for children with or at-risk for mathematics learning difficulties should be developed and evaluated through randomized-controlled studies.

Conclusions

Difficulties with mathematics are pervasive and can have lifelong consequences. Foundational number sense skills develop in early childhood and are highly predictive of mathematics achievement and difficulties. The development of number sense depends on level of representation and set size. Research suggests that number sense should be prioritized in preschool and kindergarten to provide a foundation for learning formal arithmetic and developing fluency. Overall, early number sense is critical for setting mathematics trajectories in mathematics throughout elementary school.

Implications for Parents, Service, and Policy

In contemporary educational settings, challenges in learning mathematics may go unnoticed until Grade 4. Early interventions in mathematics are less common than are those for reading, although early screening and multi-tiered intervention programs are growing as we expand our

knowledge. Preschools and kindergartens should incorporate mathematics experiences that emphasize instruction in number, number relations and number operations. The curriculum should gradually increase set size and vary type of representation.4,34 It is crucial for curriculum developers in early childhood education to concentrate on fundamental aspects of number sense. By doing so, early interventions can equip all children with the necessary foundations for success in formal mathematics.

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