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Learning Trajectories in Early Mathematics – Sequences of Acquisition and Teaching

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Introduction

Children follow natural developmental progressions in learning and development. As a simple example, children first learn to crawl, which is followed by walking, running, skipping, and jumping with increased speed and dexterity. Similarly, they follow natural developmental progressions in learning math; they learn mathematical ideas and skills in their own way. When educators understand these developmental progressions, and sequence activities based on them, they can build mathematically enriched learning environments that are developmentally appropriate and effective. These developmental paths are a main component of a *learning trajectory*.

Key Research Questions

Learning trajectories help us answer several questions.

1. What objectives should we establish?
2. Where do we start?
3. How do we know where to go next?
4. How do we get there?

Recent Research Results

Recently, researchers have come to a basic agreement on the nature of learning trajectories.[1] Learning trajectories have three parts: a) a mathematical goal; b) a developmental path along which children develop to reach that goal; and c) a set of instructional activities, or tasks, matched to each of the levels of thinking in that path that help children develop higher levels of thinking. Let's examine each of these three parts.

Goals: The Big Ideas of Mathematics

The first part of a learning trajectory is a *mathematical goal*. Our goals are the *big ideas of mathematics*—clusters of concepts and skills that are mathematically central and coherent, consistent with children’s thinking, and generative of future learning. These big ideas come from several large projects, including those from the National Council of Teachers of Mathematics and the National Math Panel.[2,3,4] For example, one big idea is that *counting can be used to find out how many are in a collection. Another would be, geometric shapes can be described, analyzed, transformed and composed and decomposed into other shapes* . It is important to realize that there are several such big ideas and learning trajectories, depending on how you classify them, there are about 12.

Development Progressions: The Paths of Learning

The second part of a learning trajectory consists of levels of thinking; each more sophisticated than the last, which lead to achieving the mathematical goal. That is, the developmental progression describes a typical path children follow in developing understanding and skill about that mathematical topic. Development of mathematics abilities begins when life begins. Young children have certain mathematical-like competencies in number, spatial sense, and patterns from birth.[5,6]

However, young children's ideas and their interpretations of situations are uniquely different from those of adults. For this reason, good early childhood teachers are careful not to assume that children "see" situations, problems, or solutions as adults do. Instead, good teachers interpret what the child is doing and thinking; they attempt to see the situation from the child's point of view. Similarly, when these teachers interact with the child, they also consider the instructional tasks and their own actions from the child's point of view. This makes early childhood teaching both demanding and rewarding.

The learning trajectories we created as part of the Building Blocks[a] and TRIAD[b] projects provide simple labels for each level of thinking in every learning trajectory. Figure 1 illustrates a part of the learning trajectory for counting. The Developmental Progression column provides both a label and description for each level, along with an example of children's thinking and behavior. It is important to note that the ages in the first column are approximate. Without experience, some children can be years behind this average age. With high-quality education, children can far exceed these averages. As an illustration, 4-year-olds in our Building Blocks curriculum meet or surpass the "5-year-old" level in most learning trajectories, including counting. (For complete learning trajectories for all topics in mathematics, see Clements & Sarama; [7] Sarama & Clements.[6] These works also review the extensive research work on which all the learning trajectories are based.).

Instructional Tasks: The Paths of Teaching

The third part of a learning trajectory consists of set of instructional tasks, matched to each of the levels of thinking in the developmental progression. These tasks are designed to help children learn the ideas and skills needed to achieve that level of thinking. That is, as teachers, we can use these tasks to promote children's growth from one level to the next. The third column in Figure 1 provides example tasks. (Again, the complete learning trajectory in Clements & Sarama, [6,7] includes not only all the developmental levels, but several instructional tasks for each level.)

Table 1. Samples from the Learning Trajectory for Counting (all examples from Clements & Sarama,[8] Clements & Sarama,[7] Sarama & Clements[6]).

Age	**Developmental Progression**	**Instructional Tasks**

1 year	**Pre-Counter** *Verbal* No verbal counting. **Chanter** *Verbal* Chants "sing-song" or sometimes-indistinguishable number words.	Associate number words with quantities and as components of the counting sequence. Repeated experience with the counting sequence in varied contexts.
2	**Reciter** *Verbal* Verbally counts with separate words, not necessarily in the correct order.	Provide repeated, frequent experience with the counting sequence in varied contexts. *Count and Race* Children verbally count along with the computer (up to 50) by adding cars to a racetrack one at a time.
3	**Reciter (10)** *Verbal* Verbally counts to ten, with *some* correspondence with objects.	*Count and Move* Have all children count from 1-10 or an appropriate number, making motions with each count. For example, say, "one" [touch head], "two" [touch shoulders], "three" [touch head], and so forth.
	Corresponder Keeps one-to-one correspondence between counting words and objects (one word for each object), at least for small groups of objects laid in a line.	*Kitchen Counter* At the computer, children click on objects one at a time while the numbers from one to ten are counted aloud. For example, they click on pieces of food and a bite is taken out of each as it is counted.

4	**Counter (Small Numbers)** Accurately counts objects in a line to 5 and answers the "how many" question with the last number counted.	*Cubes in the Box* Have the child count a small set of cubes. Put them in the box and close the lid. Then ask the child how many cubes you are hiding. If the child is ready, have him/her write the numeral. Dump them out and count together to check. *Pizza Pizzazz 2* Children count items up to 5, putting toppings on a pizza to match atarget amount.
	Producer —Counter To (Small Numbers) Counts out objects to 5. Recognizes that counting is relevant to situations in which a certain number must be placed.	*Count Motions* While waiting during transitions, have children count how many times you jump or clap, or some other motion. Then have them do those motions the same number of times. Initially, count the actions with children. *Pizza Pizzazz 3* Children add toppings to a pretend pizza (up to 5), to match target numerals.
5	**Counter and Producer (10+)** Counts and counts out objects accurately to 10, then beyond (to about 30). Has explicit understanding of cardinality (how numbers tell how many). Keeps track of objects that have and have not been counted, even in different arrangements.	*Counting Towers (Beyond 10)* To allow children to count to 20 and beyond, have them make towers with other objects such as coins. Children build a tower as high as they can, placing more coins, but not straightening coins already in the tower. The goal is to estimate and then count to find out how many coins are in your tallest tower. *Dino Shop 2* Children add dinosaurs to a box to match target numerals.

In summary, learning trajectories describe the goals of learning, the thinking and learning processes of children at various levels, and the learning activities in which they might engage. People often have several questions about learning trajectories.

Future Directions

Although learning trajectories have proven to be effective for early mathematics curricula and professional development,[9,10] there have been too few studies that have compared various ways of implementing them. Thus, their exact role remains to be studied. Also, in the early years, several learning trajectories are based on considerable research, such as those for counting and arithmetic. However, others, such as patterning and measurement, have a smaller research base. Further, there are few guidelines for many more sophisticated math topics for teaching older students. These remain challenges to the field.

Conclusions

Learning trajectories hold promise for improving professional development and teaching in the area of early mathematics. For example, the few teachers that actually led in-depth discussions in reform mathematics classrooms saw themselves not as moving through a curriculum, but as helping students move through levels of understanding.[11] Further, researchers suggest that professional development focused on learning trajectories increases not only teachers' professional knowledge but also their students' motivation and achievement.[12,13,14] Thus, learning trajectories can facilitate developmentally appropriate teaching and learning for all children.

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[a] See also the Building Blocks Web Site. Available at: http://www.ubbuildingblocks.org/. Accessed June 3, 2010.

[b] See also the TRIAD Web Site. Available at: http://www.ubtriad.org. Accessed June 3, 2010.