ENCYCLOPEDIA
on Early Childhood
Development

Learning Trajectories in Early Mathematics – Sequences of Acquisition and Teaching

Douglas H. Clements, PhD, Julie Sarama, PhD

Marsico Institute for Early Learning, College of Education, University of Denver, USA

August 2023, Éd. rév.

Introduction

Children follow natural developmental progressions in learning and development. For example, children first learn to crawl, then walk, run, skip, and jump with increasing speed and dexterity. Similarly, they follow natural developmental progressions in learning math; they learn mathematical ideas and skills in their own way.[1] When educators understand these developmental progressions, and sequence activities based on them, they can build mathematically enriched learning environments that are developmentally appropriate and effective. These developmental paths are a main component of a *learning trajectory*.

Key Research Questions

Learning trajectories help us answer several questions.

1. What goals should we establish?
2. Where do we start, that is, what are children's developmental level?

©2023-2026 ABILIO | NUMERACY

3. How do we know where to go next?
4. How do we get there?

Recent Research Results

Recently, researchers have come to a basic agreement on the nature of learning trajectories.2 Learning trajectories have three parts: a) a mathematical goal; b) a developmental path along which children develop to reach that goal; and c) a set of instructional activities, matched to each of the levels of thinking in that path, that help children develop higher levels of thinking. Let's examine each of these three parts.

Goals: The Big Ideas of Mathematics

The first part of a learning trajectory is a *mathematical goal*. Goals involve the *big ideas of mathematics*—clusters of concepts and skills that are mathematically central and coherent, consistent with children's thinking, and generative of future learning. These big ideas come from several national efforts.[3-6] For example, one big idea is that *counting can be used to find out how many are in a collection. Another would be, geometric shapes can be described, analyzed, transformed and composed and decomposed into other shapes.* It is important to realize that there are several such big ideas and learning trajectories.

Development Progressions: The Paths of Learning

The second part of a learning trajectory consists of levels of thinking; each more sophisticated than the last, through which most children progress on their way to achieving the mathematical goal. That is, the developmental progression describes a typical path children follow in developing understanding and skill about that mathematical topic. Development of mathematics abilities begins when life begins. Young children have certain mathematical-like competencies in number, spatial sense, and patterns from birth.[1,4]

However, young children's ideas and their interpretations of situations are uniquely different from those of adults. For this reason, early childhood teachers are careful not to assume that children "see" situations, problems, or solutions as they do. Instead, teachers interpret what the child is doing and thinking, they attempt to see the situation from the child's point of view. Similarly, when these teachers interact with the child, they also consider the instructional activities and their own actions through the child's eyes. This makes early childhood teaching

©2023-2026 ABILIO | NUMERACY

both demanding and rewarding.

Learning trajectories provide simple labels and descriptions for each level of thinking in every mathematical topic. Figure 1 illustrates a part of the learning trajectory for counting. The Developmental Progression column provides both a label and description for each level. It is important to note that the ages in the first column are approximate. Without experience, some children can be years behind this average age. With high-quality education, children can far exceed these averages. [For complete learning trajectories for all topics, including the research on which they are based, see references[1,7,] as well as LearningTrajectorie.org].

Instructional Activities: The Paths of Teaching

The third part of a learning trajectory consists of set of instructional strategies and activities, matched to each of the levels of thinking in the developmental progression, designed to help children learn the ideas and skills needed to achieve that level of thinking. That is, as teachers, we can use these strategies and activities to promote children's growth from one level to the next. The third column in Figure 1 provides examples.

Table 1. Sample Levels from the Learning Trajectory for Counting [From references[1,7,] as well as LearningTrajectorie.org]

Age	**Developmental Progression**	**Instructional Activities**
1 year	**Number Word Sayer: Foundations.** No verbal counting but names some number words. **Chanter** Chants in singsong fashion or sometimes-indistinguishable number words.	Associate number words with small quantities (see “Subitizing” in the resources) and verbally count for fun (e.g., going up stairs).
2	**Reciter** Verbally counts with separate words, not necessarily in the correct order.	Provide repeated, frequent experience with the counting sequence in varied contexts. *Count and Race*. Children verbally count along with the computer (up to 50) by adding cars to a racetrack one at a time.

©2023-2026 ABILIO | NUMERACY

Age	Developmental Progression	Instructional Activities
3	**Reciter (10)** Verbally counts to ten, with *some* correspondence with objects.	*Count and Move.* Have all children count from 1-10 or an appropriate number, making motions with each count. For example, say, “one” [touch head], “two” [touch shoulders], “three” [touch head], and so forth.
	Corresponder Keeps one-to-one correspondence between counting words and objects (one word for each object), at least for small groups of objects laid in a line.	*Counting Wand.* Children use a counting wand to count the number of children in a group, focusing on the 1-to-1 correspondence.
4	**Counter (Small Numbers)** Accurately counts objects in a line to 5 and answers the “how many” question with the last number counted.	*Cubes in the Box.* Have the child count a small set of cubes. Put them in the box and close the lid. Then ask the child how many cubes you are hiding. If the child is ready, have him/her write the numeral. Dump them out and count together to check. *Pizza Pizzazz 2* Children count items up to 5, putting toppings on a pizza to match a target amount.
	Producer —Counter To (Small Numbers) Counts out objects to 5. Recognizes that counting is relevant to situations in which a certain number must be placed.	*Count Motions.* While waiting during transitions, have children count how many times you jump or clap, or some other motion. Then have them do those motions the same number of times. Initially, count the actions with children.

©2023-2026 ABILIO | NUMERACY

Age	**Developmental Progression**	**Instructional Activities**
5	**Counter and Producer (10+)** Counts and counts out objects accurately to 10, then beyond (to about 30). Has explicit understanding of cardinality (how numbers tell how many). Keeps track of objects that have and have not been counted, even in different arrangements.	*Counting Towers (Beyond 10).* To allow children to count to 20 and beyond, have them make towers with other objects such as coins. Children build a tower as high as they can, placing more coins, but not straightening coins already in the tower. The goal is to estimate and then count to find out how many coins are in your tallest tower.

In summary, learning trajectories describe the goals of learning, the thinking and learning processes of children at various levels, and the learning activities in which they might engage. People often have several questions about learning trajectories.

How Do Learning Trajectories' Developmental Levels Support Teaching and Learning? The levels help teacher understand children's thinking; create, modify, or sequence activities. Teachers who understand learning trajectories are more effective and efficient and engage children in mathematics joyfully. Through planned teaching and also by encouraging informal, incidental math, teachers help children learn at an appropriate and deep level.

There are Ages in the Learning Trajectories. Should I Plan to Help Children Develop Just the Levels that Correspond to my Children's Ages? The ages in the table are typical ages at which children develop these ideas. But these are rough guides only—children differ widely. Furthermore, the children achieve much later levels with high-quality education. So, these are approximate levels to help orient educators not goals. Children who are provided high-quality math experiences are capable of developing to levels one or more years beyond their peers.

Are the Instructional Activities the Only Way to Teach Children to Achieve Higher Levels of Thinking? No, there are many ways. In some cases, however, there is some research evidence that these are especially effective ways. In other cases, they are simply illustrations of the kind of activity that would be appropriate to reach that level of thinking. Further, teachers need to use a variety of pedagogical strategies in teaching the content, presenting the activities, guiding children in completing them, and so forth.

©2023-2026 ABILIO | NUMERACY

Future Directions

Although learning trajectories have proven to be effective for early mathematics curricula and professional development8-10, much remains to be studied, such as learning trajectories for older students. Also, in the early years, several learning trajectories are based on considerable research, such as those for counting and arithmetic. However, others, such as patterning have a smaller research base. These remain challenges to the field.

Conclusions

Learning trajectories hold promise for improving professional development and teaching in the area of early mathematics.8,11,12 Further, researchers suggest that professional development focused on learning trajectories increases not only teachers' professional knowledge but also their students' motivation and achievement.9,13-15 Thus, learning trajectories can facilitate engaging and developmentally appropriate teaching and learning for all children.

Author Note:

This research was supported by the Institute of Education Sciences, U.S. Department of Education through Grants R305A120813, R305K05157, and R305A110188. The opinions expressed are those of the authors and do not represent views of the U.S. Department of Education.

References

1. Clements DH, Sarama J. *Learning and teaching early math: The learning trajectories approach*. 3rd ed. New York, NY: Routledge; 2020.

2. Maloney AP, Confrey J, Nguyen KH, eds. *Learning over time: Learning trajectories in mathematics education*. New York, NY: Information Age Publishing; 2014.

3. Clements DH, Sarama J, DiBiase A-M. *Engaging young children in mathematics: Standards for early childhood mathematics education*. Mahwah, NJ: Lawrence Erlbaum Associates Publishers; 2004.

©2023-2026 ABILIO | NUMERACY

4. Cross CT, Woods TA, Schweingruber H. *Mathematics learning in early childhood: Paths toward excellence and equity*. Washington DC: National Research Council of the National Academics; 2009.

5. NCTM. *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics; 2000.

6. NGA/CCSSO. *Common core state standards.* Washington, DC: National Governors Association Center for Best Practices, Council of Chief State School Officers; 2010.

7. Sarama J, Clements DH. *Early childhood mathematics education research: Learning trajectories for young children*. New York, NY: Routledge; 2009.

8. Clements DH, Sarama J, Layzer C, Unlu F. Implementation of a scale-up model in early childhood: Long-term impacts on mathematics achievement. *Journal for Research in Mathematics Education*. 2023;54(1):64-88.

9. Sarama J, Clements DH, Guss SS. Longitudinal evaluation of a scale-up model for professional development in early mathematics. In: Dunekacke S, Jegodtka A, Koinzer T, Eilerts K, Jenßen L, eds. *Early childhood teachers' professional competence in mathematics* . London, England: Routledge: 2022:163-186.

10. Dumas DG, McNeish D, Sarama J, Clements DH. Preschool mathematics intervention can significantly improve student learning trajectories through elementary school. *AERA Open*. 2019;5(4):1-5.

11. Clements DH, Sarama J, Baroody AJ, Joswick C. Efficacy of a learning trajectory approach compared to a teach-to-target approach for addition and subtraction. *ZDM Mathematics Education*. 2020;52(4):637-648.

12. Verschaffel L, Bojorquea G, Torbeyns J, Van Hoof J. Persistence of the Building Blocks' impact on Ecuadorian children's early numerical abilities. In: *Proceedings of the EARLI 2019*. Aachen University, Germany:10-16.

©2023-2026 ABILIO | NUMERACY

13. Clarke BA. A shape is not defined by its shape: Developing young children’s geometric understanding. *Journal of Australian Research in Early Childhood Education.* 2004;11(2):110-127.

14. Fennema EH, et al. A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*. 1996;27:403-434.

15. Wright RJ, Martland J, Stafford AK, Stanger G. *Teaching number: Advancing children's skills and strategies*. London: Sage Publications; 2002.

©2023-2026 ABILIO | NUMERACY